Complex Systems Made Simple

1. Introduction

2. A Complex Systems Sampler

   a. Cellular automata

   b. Pattern formation

   c. Swarm intelligence

   d. Complex networks:
      - ***Three network metrics***
      - ***Random & regular networks***
      - ***Small-world & scale-free networks***
      - ***Case studies***

   e. Spatial communities

   f. Structured morphogenesis

3. Commonalities

4. NetLogo Tutorial

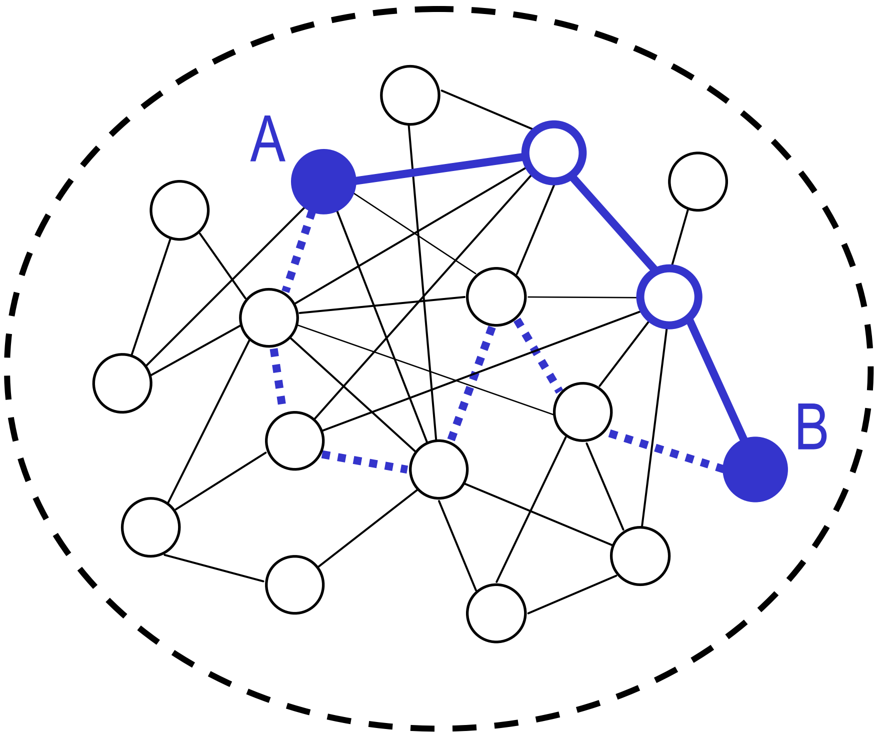
2. A Complex Systems Sampler

d. Complex networks

- ✓ complex networks are the backbone of complex systems
  - every complex system is a network of interaction among numerous smaller elements
  - some networks are geometric or regular in 2-D or 3-D space
  - other contain "long-range" connections or are not spatial at all
  - understanding a complex system = break down into parts + reassemble
- ✓ network anatomy is important to characterize because structure affects function (and vice-versa)
- ✓ ex: structure of social networks
  - prevent spread of diseases
  - control spread of information (marketing, fads, rumors, etc.)
- ✓ ex: structure of power grid / Internet
  - understand robustness and stability of power / data transmission

2. A Complex Systems Sampler

d. Complex networks – *Three metrics: average path length*



The path length between A and B is 3

- the *path length* between two nodes A and B is the smallest number of edges connecting them:

$$l(A, B) = \min l(A, A_i, \dots A_n, B)$$

- the *average path length* of a network over all pairs of N nodes is

$$L = \langle l(A, B) \rangle$$

$$= 2/N(N-1) \sum_{A,B} l(A, B)$$

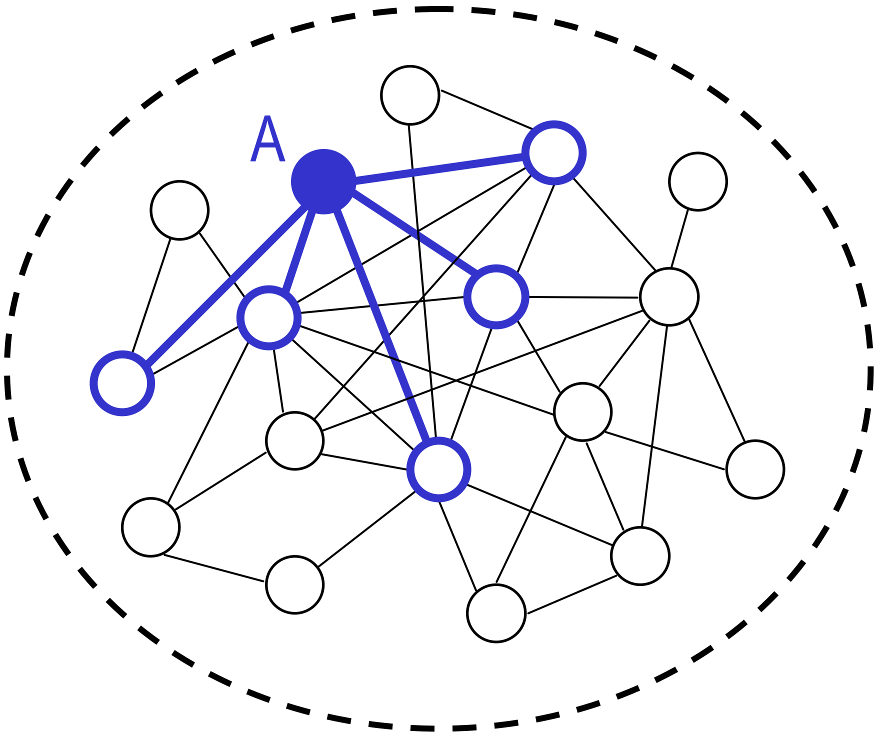
- the *network diameter* is the maximal path length between two nodes:

$$D = \max l(A, B)$$

- property: $1 \leq L \leq D \leq N-1$

2. A Complex Systems Sampler

d. Complex networks – *Three metrics: degree distribution*

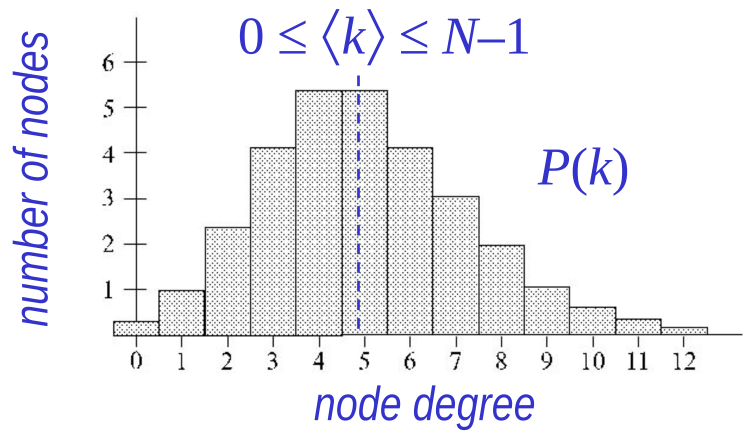


The degree of A is 5

- the *degree* of a node A is the number of its connections (or neighbors), k_A
- the *average degree* of a network is

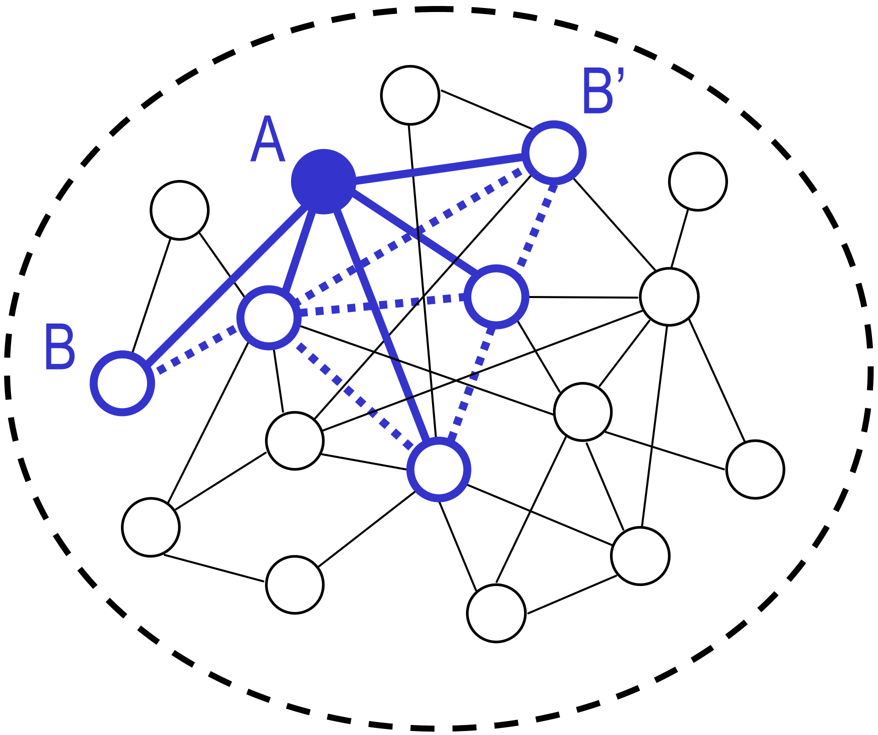
$$\langle k \rangle = 1/N \sum_A k_A$$

- the *degree distribution* function $P(k)$ is the histogram (or probability) of the node degrees: it shows their spread around the average value



2. A Complex Systems Sampler

d. Complex networks – *Three metrics: clustering coefficient*



The clustering coefficient of A is 0.6

- the *neighborhood* of a node A is the set of k_A nodes at distance 1 from A
- given the number of *pairs* of neighbors:

$$F_A = \sum_{B,B'} 1$$
$$= k_A (k_A - 1) / 2$$

- and the number of pairs of neighbors that are also *connected* to each other:

$$E_A = \sum_{B \leftrightarrow B'} 1$$

- the *clustering coefficient* of A is

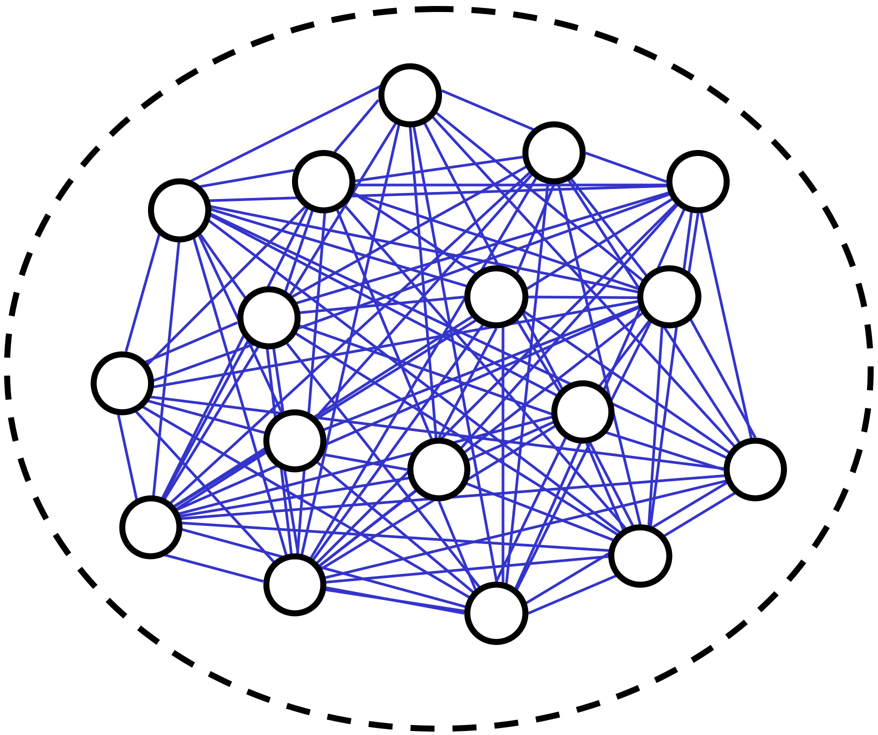
$$C_A = E_A / F_A \;\; \leq 1$$

- and the *network clustering coefficient*:

$$\langle C \rangle = 1/N \sum_A C_A \;\; \leq 1$$

2. A Complex Systems Sampler

d. Complex networks – *Regular networks: fully connected*



A fully connected network

- in a *fully (globally) connected* network, each node is connected to all other nodes
- fully connected networks have the *LOWEST path length* and diameter:

$$L = D = 1$$

- the *HIGHEST clustering coefficient*:

$$C = 1$$

- and a *PEAK degree distribution* (at the largest possible constant):

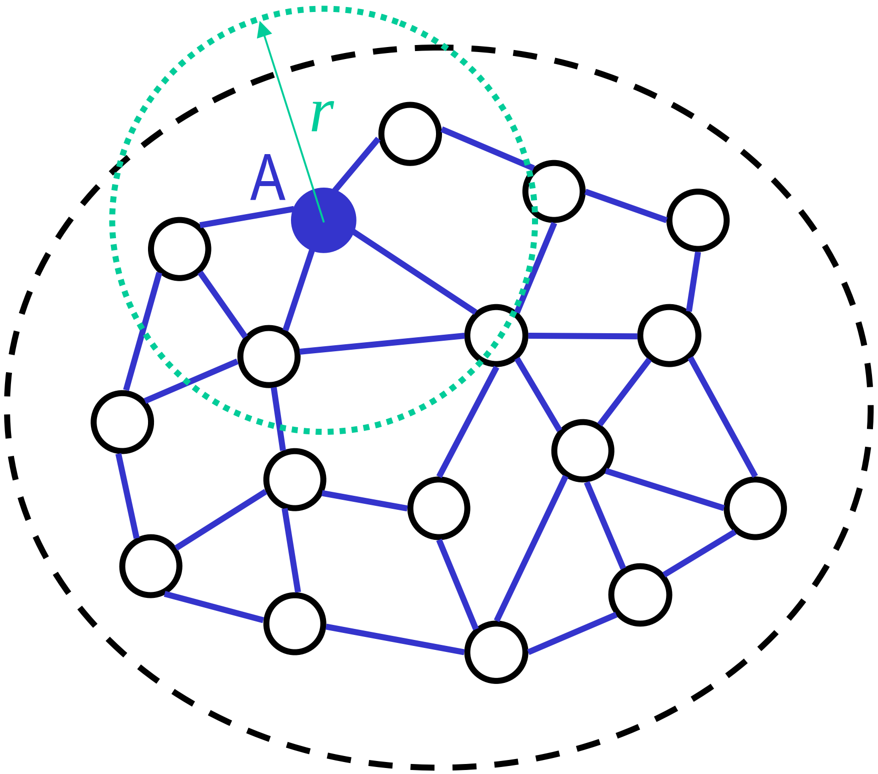
$$k_A = N-1, \quad P(k) = \delta(k - N+1)$$

- also the highest number of edges:

$$E = N(N-1)\,/\,2 \;\sim\; N^2$$

2. A Complex Systems Sampler

d. Complex networks – *Regular networks: lattice*



A 2-D lattice network

- a *lattice* network is generally structured against a geometric 2-D or 3-D background
- for example, each node is connected to its nearest neighbors depending on the Euclidean distance:

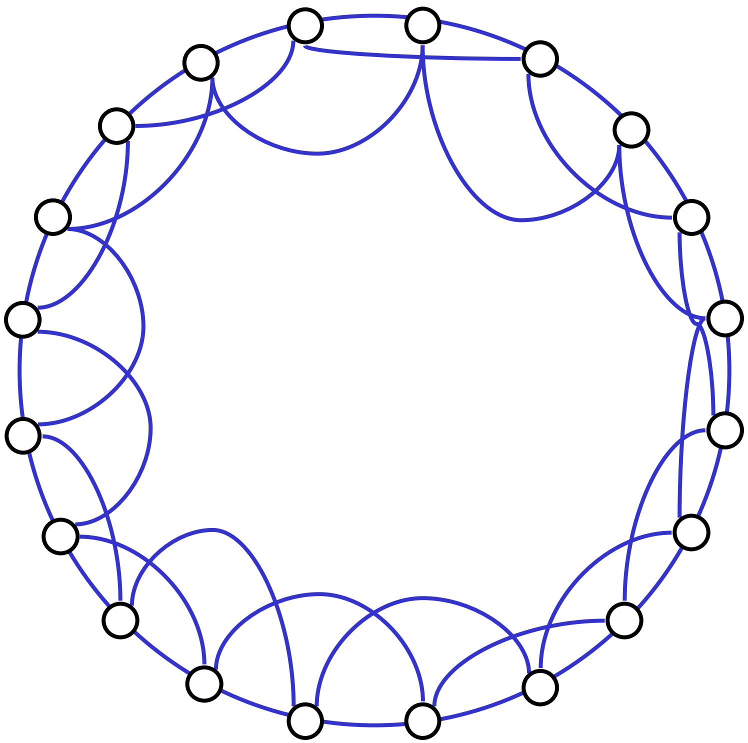
$$A \leftrightarrow B \iff d(A, B) \leq r$$

- the radius r should be sufficiently small to remain far from a fully connected network, i.e., keep a large diameter:

$$D >> 1$$

2. A Complex Systems Sampler

d. Complex networks – *Regular networks: lattice: ring world*



A ring lattice with K = 4

- in a *ring lattice*, nodes are laid out on a circle and connected to their K nearest neighbors, with $K << N$
- *HIGH average path length*:

$$L \approx N / 2K \quad \sim N \quad \text{for } N >> 1$$

(mean between closest node $l = 1$ and antipode node $l = N / K$)

- *HIGH clustering coefficient*:

$$C \approx 0.75 \quad \text{for } K >> 1$$

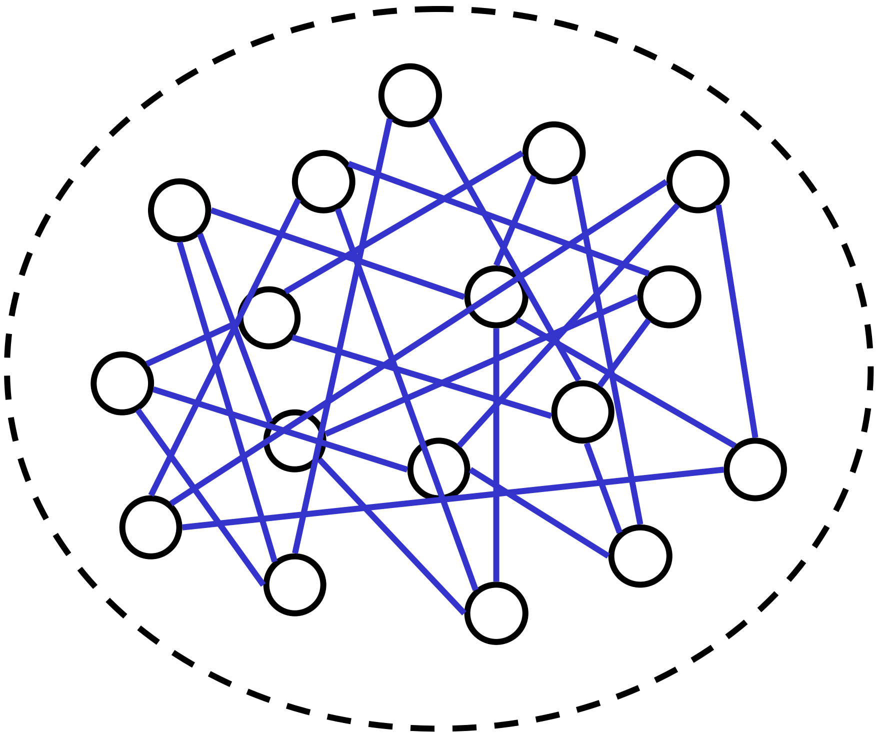
(mean between center with K edges and farthest neighbors with $K/2$ edges)

- *PEAK degree distribution* (low value):

$$k_A = K, \quad P(k) = \delta(k - K)$$

2. A Complex Systems Sampler

d. Complex networks – *Random networks*



A random graph with $p = 3/N = 0.18$

- in a *random graph* each pair of nodes is connected with probability p
- *LOW average path length*:

  $L \approx \ln N / \ln\langle k\rangle \sim \ln N$ for $N >> 1$

  (because the entire network can be covered in about L steps: $N \sim \langle k\rangle^L$)
- *LOW clustering coefficient* (if sparse):

  $C = p = \langle k\rangle / N << 1$ for $p << 1$

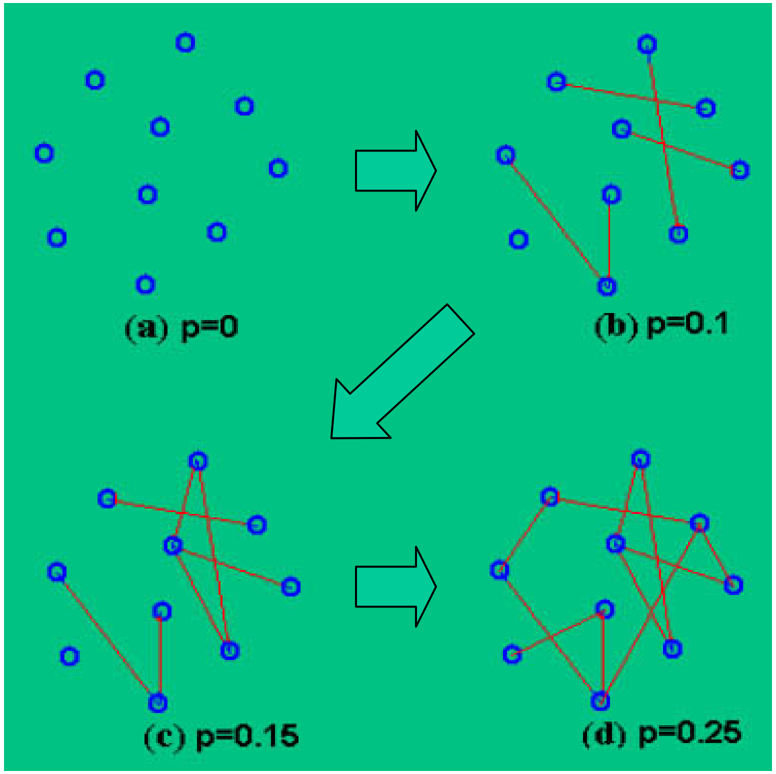
  (because the probability of 2 neighbors being connected is p, by definition)
- *PEAK (Poisson) degree distribution* (low value):

  $\langle k\rangle \approx pN, \quad P(k) \approx \delta(k - pN)$

2. A Complex Systems Sampler

d. Complex networks – *Random networks*

NetLogo model: /Networks/Giant Component



Percolation in a random graph

(Wang, X. F., 2002)

- Erdős & Rényi (1960): above a critical value of random connectivity the network is almost certainly connected in one single component
- *percolation* happens when "picking one button (node) will lift all the others"
- the critical value of probability p is

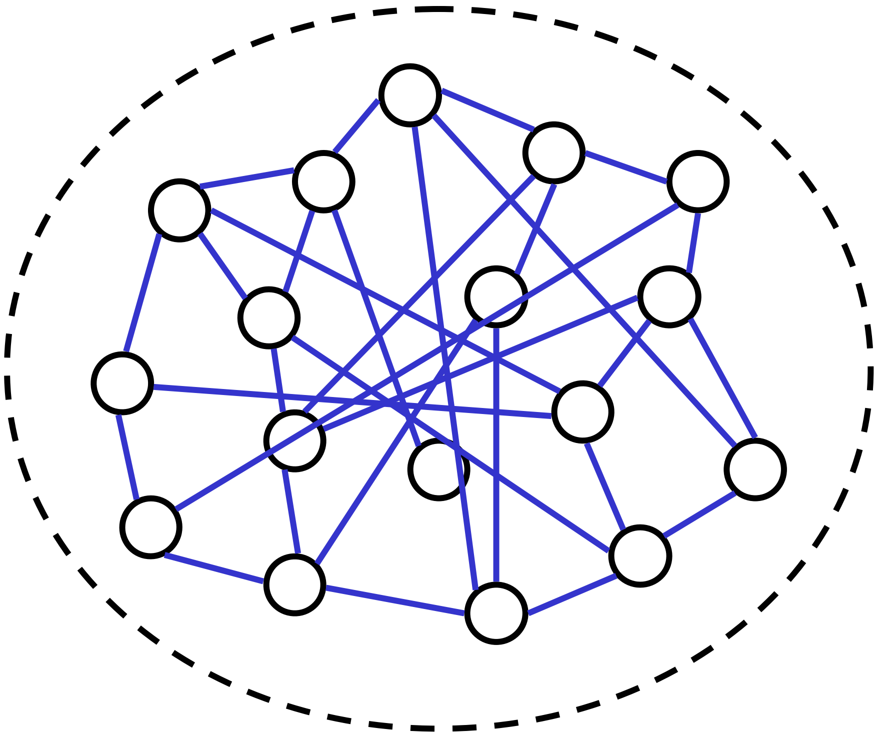
$$p_c \approx \ln N / N$$

- and the corresponding average critical degree:

$$\langle k_c \rangle \approx p_c N \approx \ln N$$

2. A Complex Systems Sampler

d. Complex networks – *Small-world networks*



A Watts-Strogatz small-world network

- a network with *small-world EFFECT* is ANY large network that has a low average path length:

$$L << N \quad \text{for } N >> 1$$

- famous "6 degrees of separation"
- the *Watts-Strogatz (WS) small-world MODEL* is a hybrid network between a regular lattice and a random graph
- WS networks have both the LOW average path length of random graphs:

$$L \sim \ln N \quad \text{for } N >> 1$$

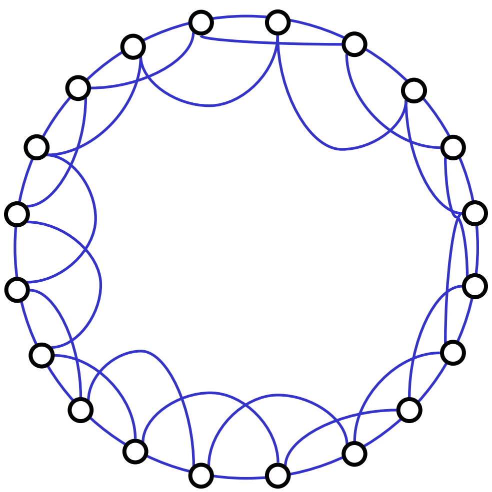
- and the HIGH clustering coefficient of regular lattices:

$$C \approx 0.75 \quad \text{for } K >> 1$$

2. A Complex Systems Sampler

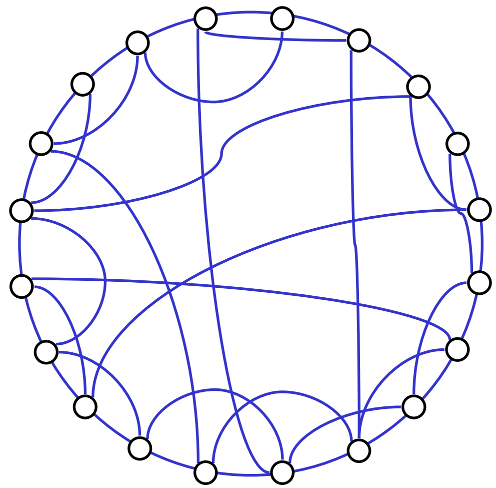
d. Complex networks – ***Small-world networks***

Ring Lattice
- large world
- well clustered



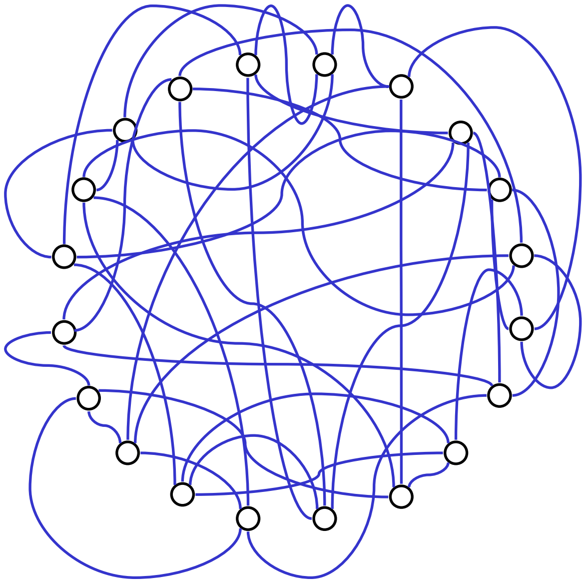
$p = 0$ (order)

Watts-Strogatz (1998)
- small world
- well clustered



$0 < p < 1$

Random graph
- small world
- poorly clustered

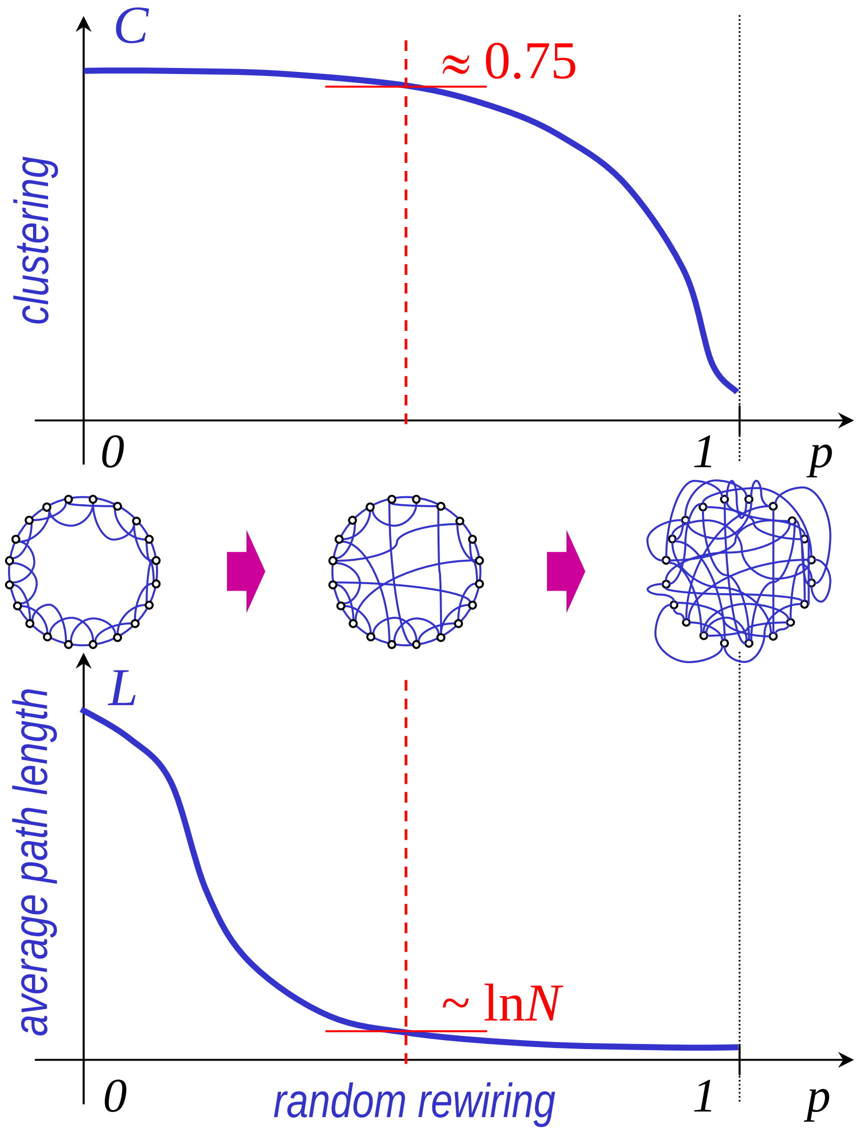


$p = 1$ (disorder)

➢ the WS model consists in gradually rewiring a regular lattice into a random graph, with a probability p that an original lattice edge will be reassigned at random

2. A Complex Systems Sampler

d. Complex networks – *Small-world networks*

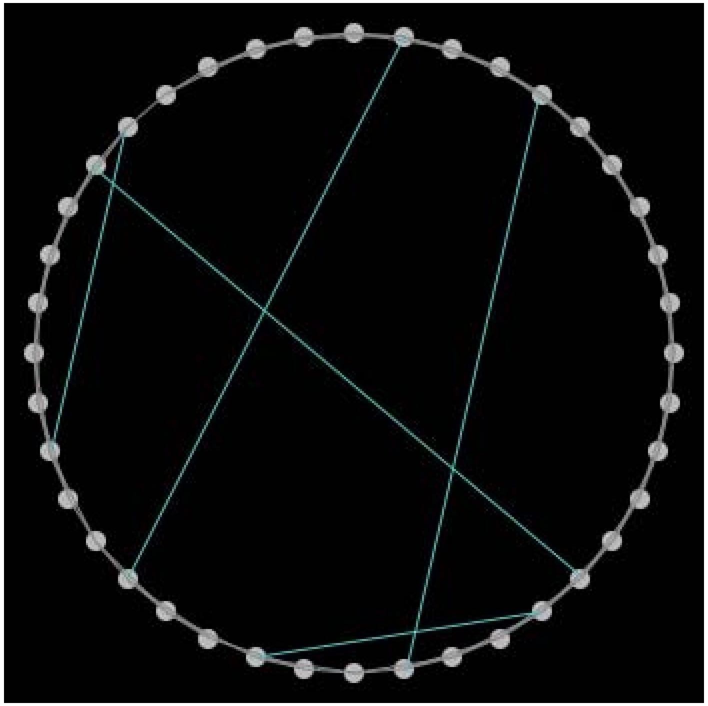


- the clustering coefficient is resistant to rewiring over a broad interval of p
  - it means that the small-world effect is hardly detectable locally: nodes continue seeing mostly the same “clique” of neighbors
- on the other hand, the average path length drops rapidly for low p
  - as soon as a few long-range “short-cut” connections are introduced, the original large-world starts collapsing
  - through a few bridges, far away cliques are put in contact and this is sufficient for a rapid spread of information

2. A Complex Systems Sampler

d. Complex networks – ***Small-world networks***

NetLogo model: /Networks/Small Worlds

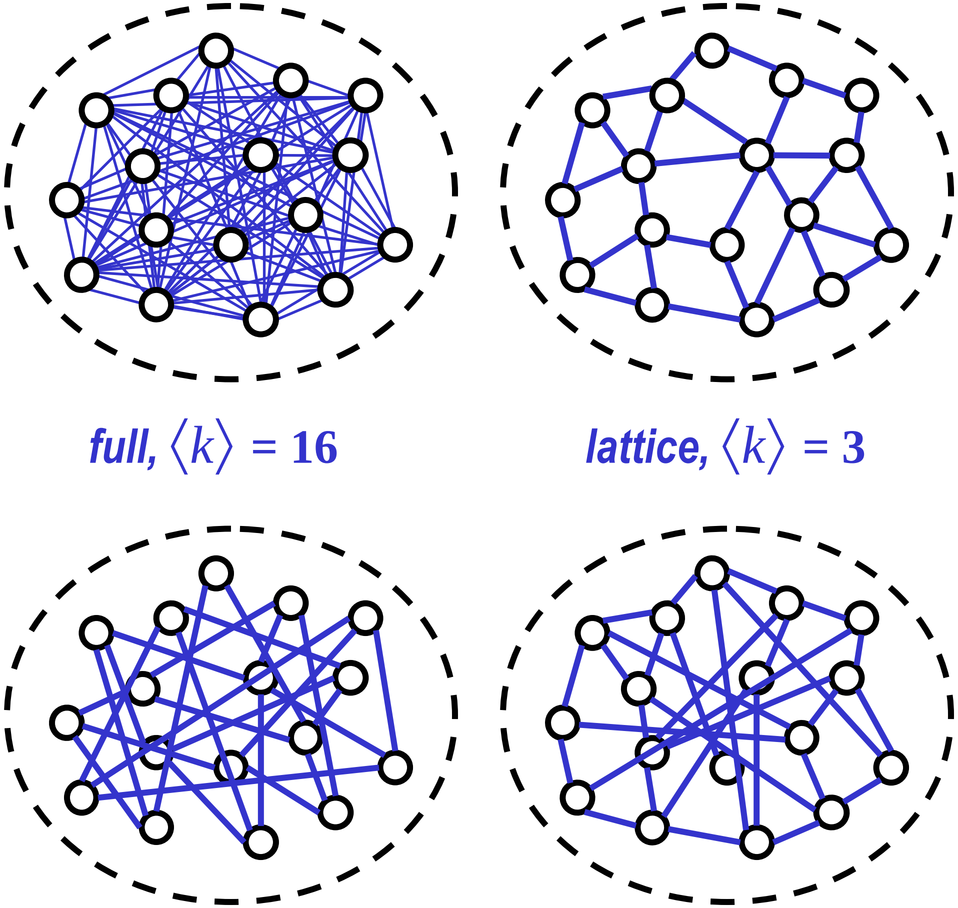


Modeling & simulation

- setup:
  - number of nodes
  - each node is connected to its 2+2 nearest neighbors
- rewire ONCE:
  - rewire *one* ring lattice *incrementally* (ignoring the rewiring probability)
- rewire ALL:
  - rewire *several* ring lattices *in one shot* under a certain rewiring probability
- calculate 2 metrics for each network:
  - average path length
  - clustering coefficient

2. A Complex Systems Sampler

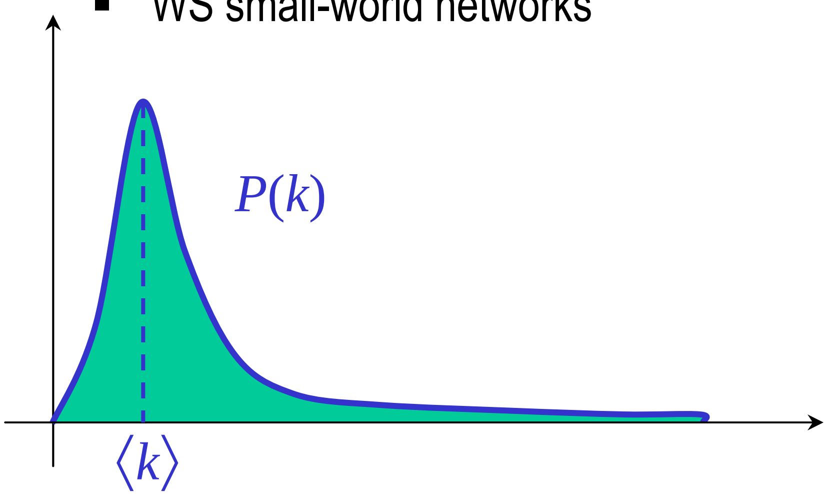
d. Complex networks – *Small-world networks*



full, $\langle k \rangle = 16$ *lattice,* $\langle k \rangle = 3$

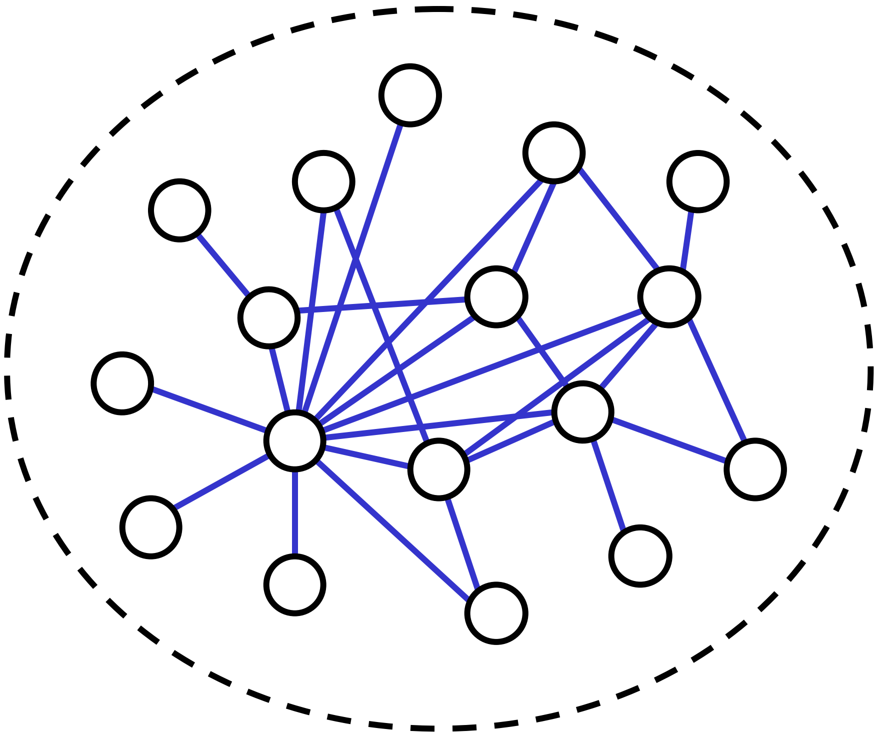
random, $\langle k \rangle = 3$ *WS small-world,* $\langle k \rangle = 3$

- on the other hand, the WS model still has a PEAK (Poisson) degree distribution (uniform connectivity)
- in that sense, it belongs to the same family of *exponential networks*:
  - fully connected networks
  - lattices
  - random graphs
  - WS small-world networks



2. A Complex Systems Sampler

d. Complex networks – ***Scale-free networks***

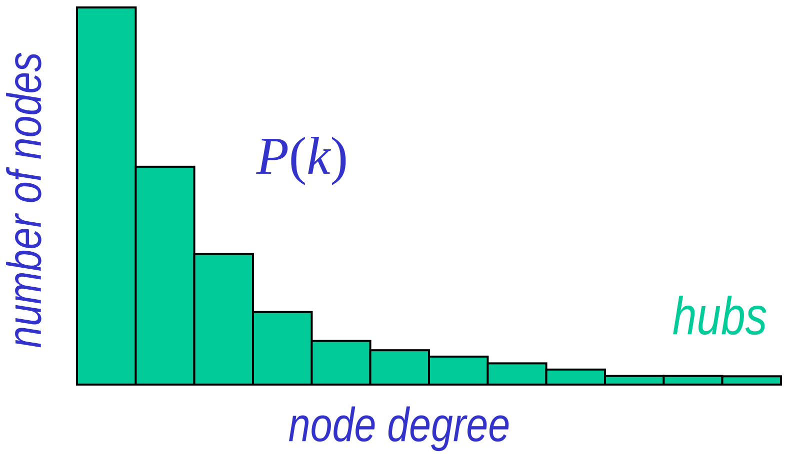


A schematic scale-free network

- in a *scale-free network* the degree distribution follows a POWER-LAW:

$$P(k) \sim k^{-\gamma}$$

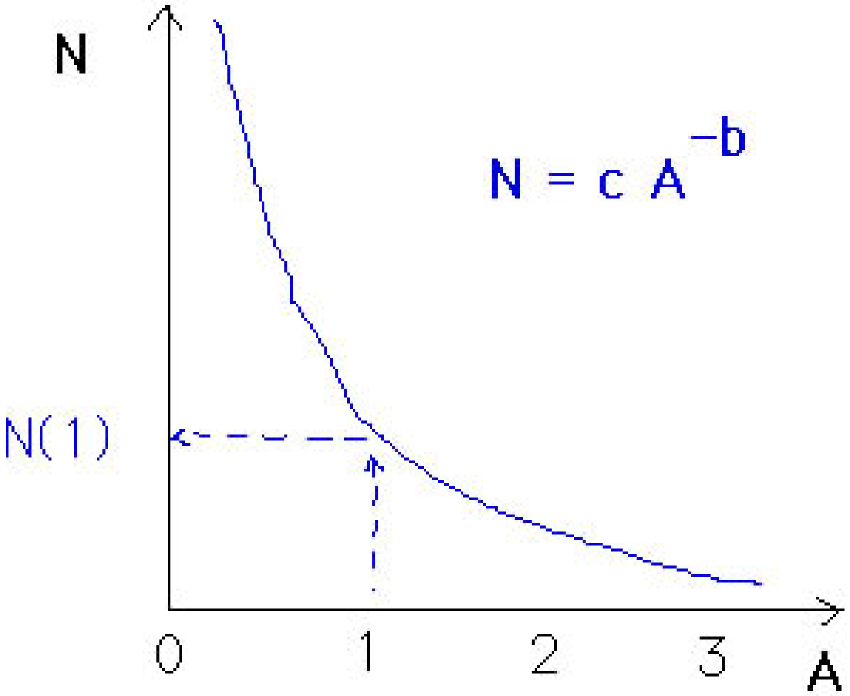
- there exists a small number of highly connected nodes, called *hubs* (tail of the distribution)
- the great majority of nodes have few connections (head of the distribution)

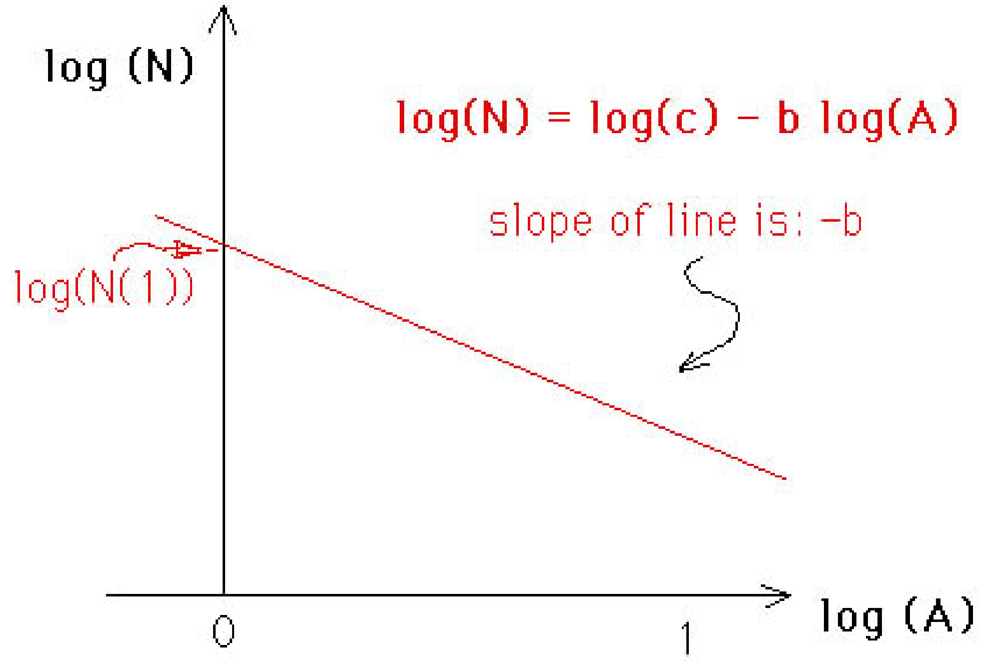


2. A Complex Systems Sampler

d. Complex networks – ***Scale-free networks***

- hyperbola-like, in linear-linear plot
- straight line, in log-log plot





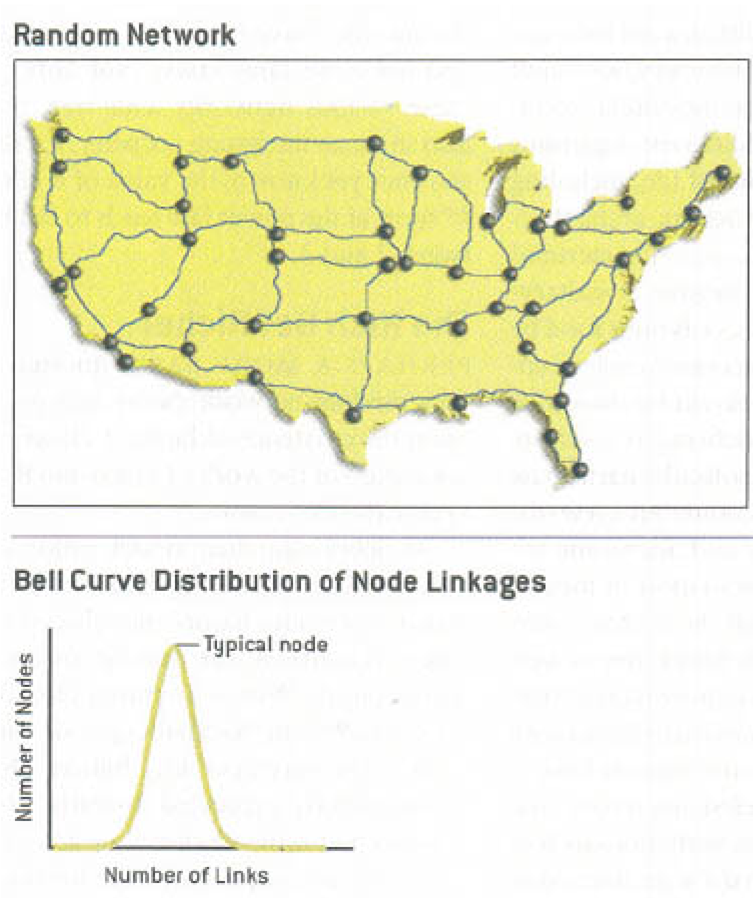
Typical aspect of a power law

(image from Larry Ruff, University of Michigan, http://www.geo.lsa.umich.edu/~ruff)

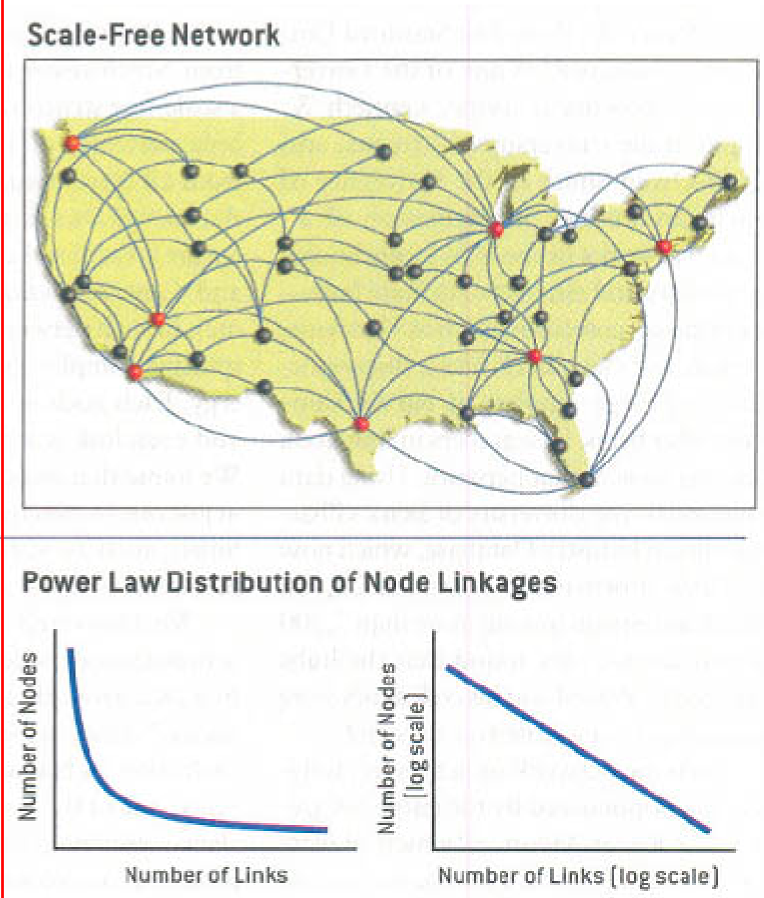
2. A Complex Systems Sampler

d. Complex networks – ***Scale-free networks***

U.S. highway system



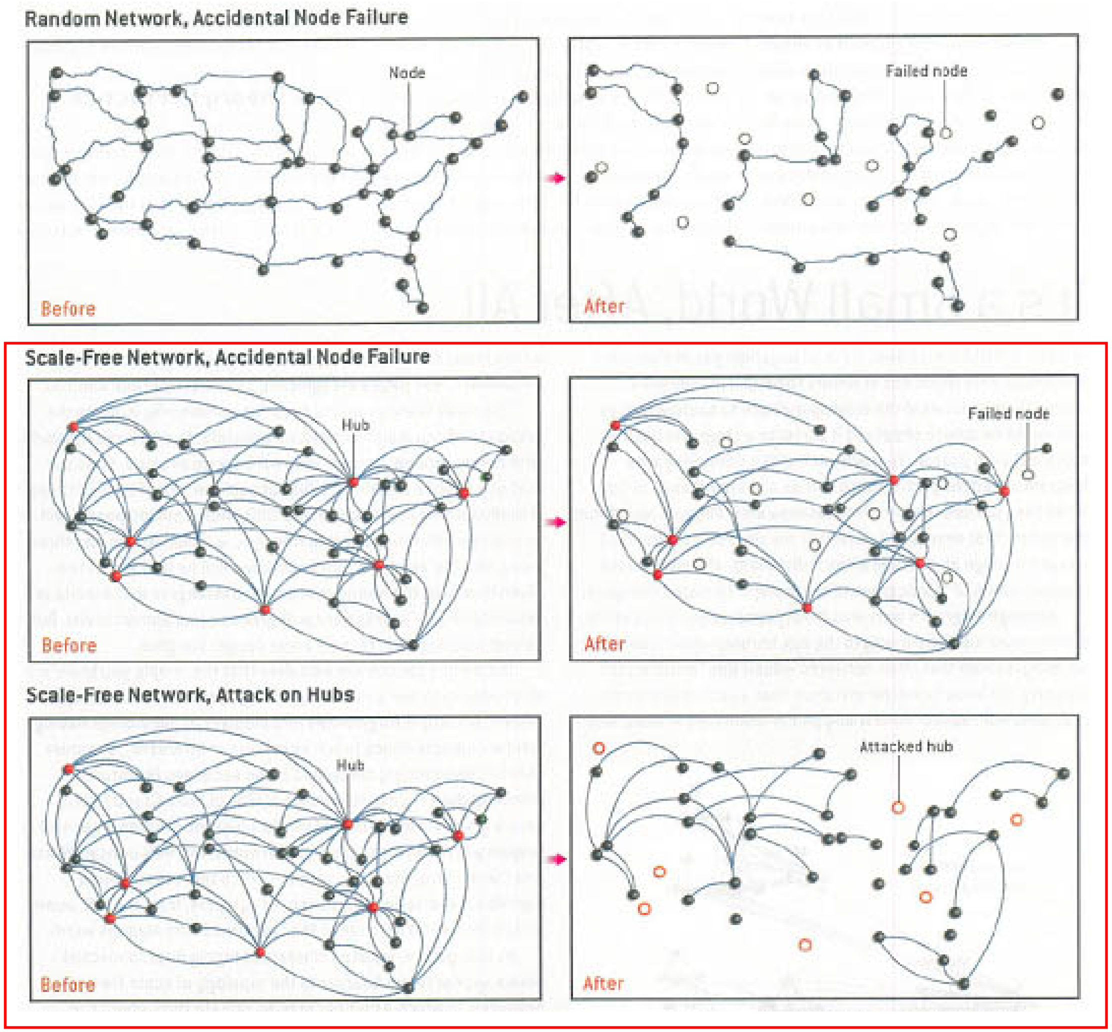
U.S. airline system



(Barabási & Bonabeau, 2003)

2. A Complex Systems Sampler

d. Complex networks – *Scale-free networks*



Effect of failures and attacks on scale-free networks

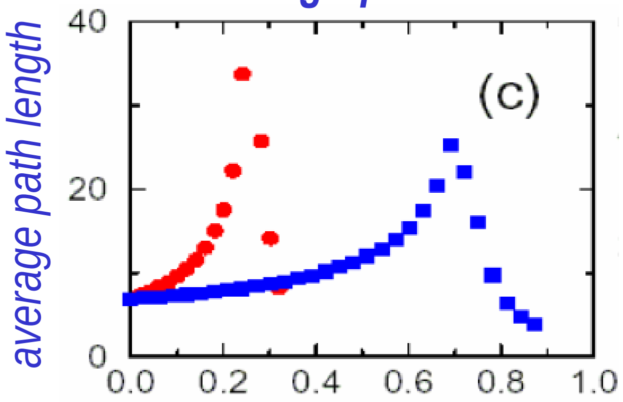
(Barabási & Bonabeau, 2003)

- regular networks are not resistant to random node failures: they quickly break down into isolated fragments
- scale-free networks are remarkably resistant to random accidental node failures . . .
- . . . however they are also highly vulnerable to targeted attacks on their hubs

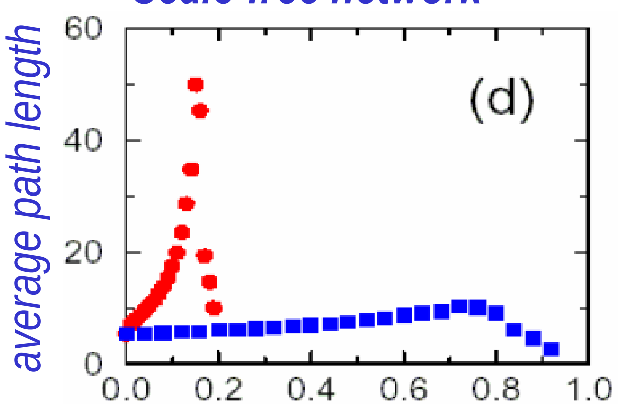
2. A Complex Systems Sampler

d. Complex networks – ***Scale-free networks***

Random graph



Scale-free network



random and targeted removal of nodes

(Albert, Jeong & Barabási, 1999)

- in a random graph the average path length increases significantly with node removal, then eventually breaks down

→ *for a while, the network becomes a large world*

- in a scale-free network, the average path length is preserved during ***random node removal***

→ *it remains a small world*

- however, it fails even faster than a random graph under ***targeted removal***

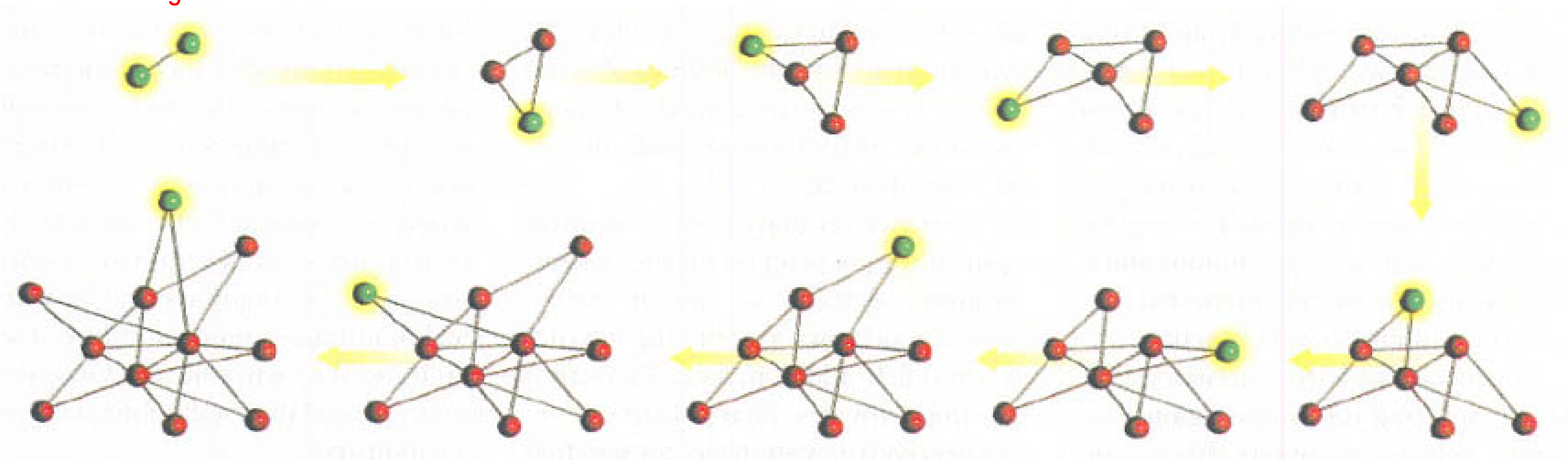
2. A Complex Systems Sampler

d. Complex networks – ***Scale-free networks***

- the *Barabási-Albert model*, reproduces the scale-free property by:
  - growth and
  - (linear) preferential attachment

- growth: a node is added at each step
- attachment: new nodes tend to prefer well-connected nodes ("the rich get richer" or "first come, best served") in linear proportion to their degree

NetLogo model: /Networks/Preferential Attachment



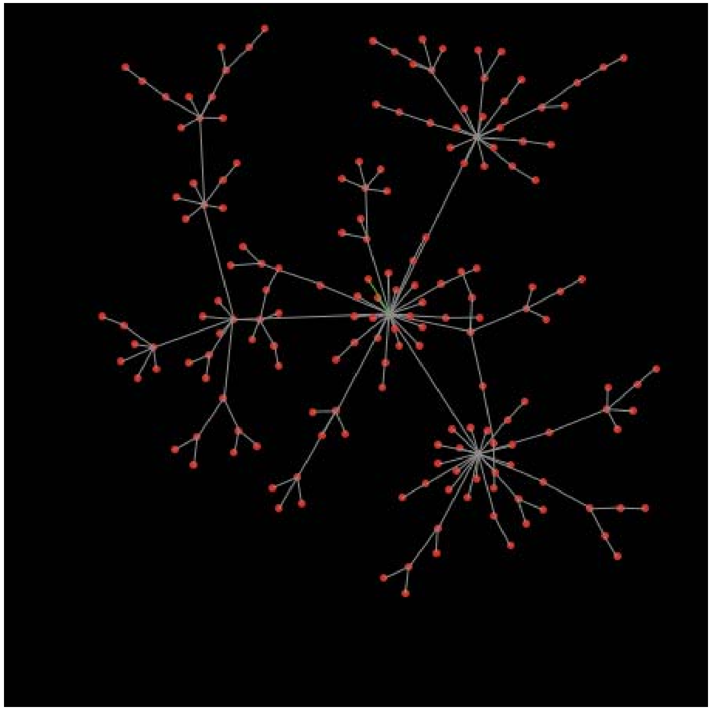
Growth and preferential attachment creating a scale-free network

(Barabási & Bonabeau, 2003)

2. A Complex Systems Sampler

d. Complex networks – *Scale-free networks*

NetLogo model: /Networks/Preferential Attachment

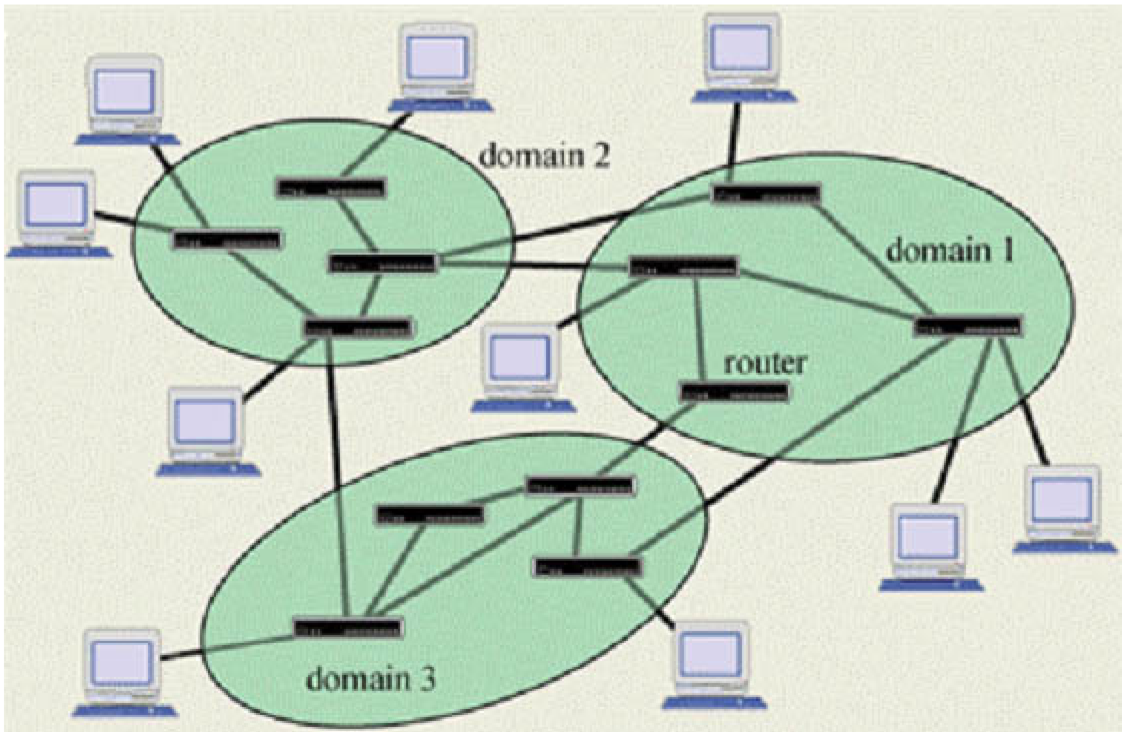


Modeling & simulation

- setup:
  - 2 nodes, 1 link
- step:
  - a new node is added to the network and preferrentially attached to _one_ other node
- calculate 1 metric:
  - degree distribution (displayed as histogram and in log-log coordinates)

2. A Complex Systems Sampler

d. Complex networks – *Case studies: Internet*



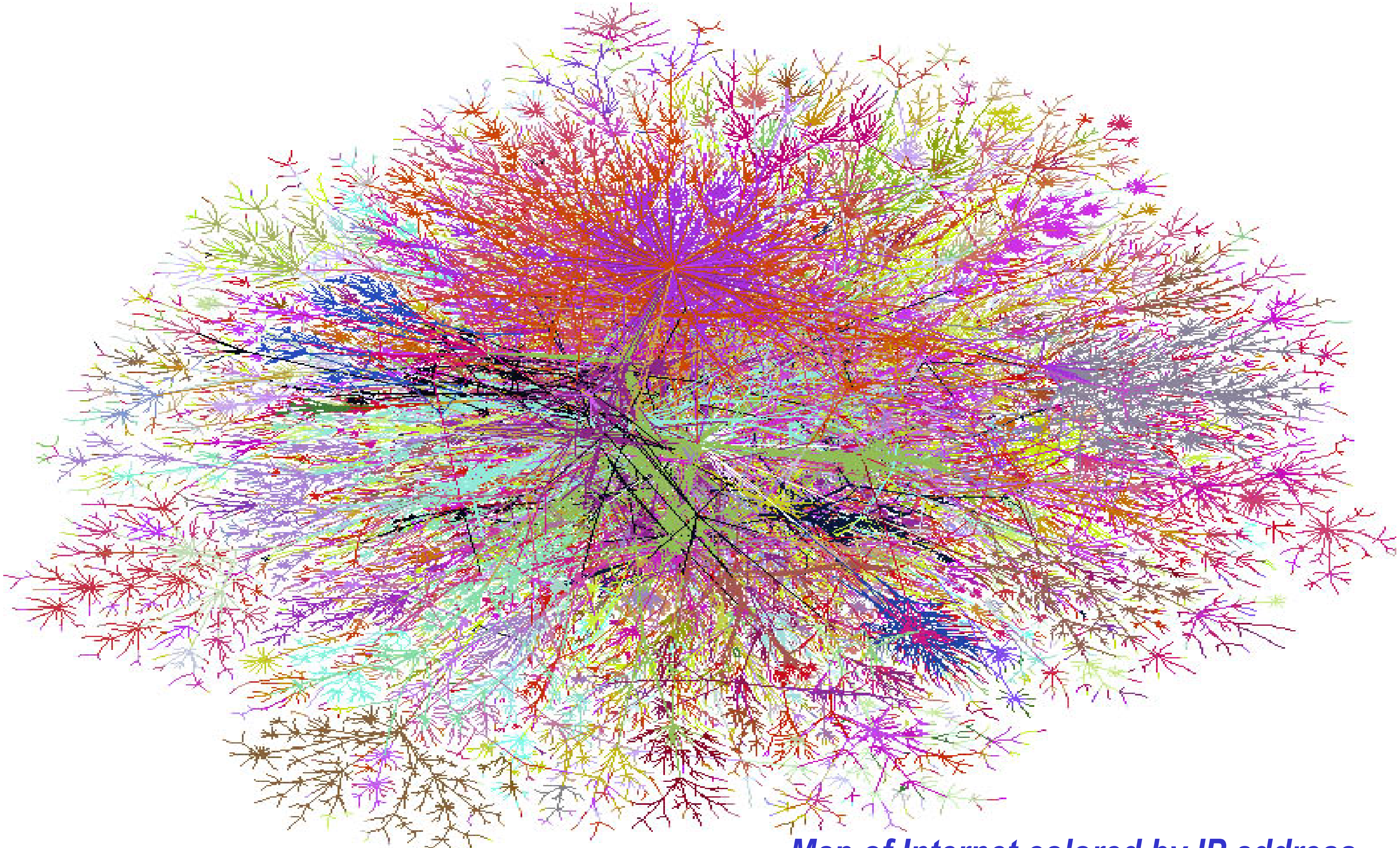
Schema of the Internet

(Wang, X. F., 2002)

- the Internet is a network of routers that transmit data among computers
- routers are grouped into domains, which are interconnected
- to map the connections, "traceroute" utilities are used to send test data packets and trace their path

2. A Complex Systems Sampler

d. Complex networks – *Case studies: Internet*

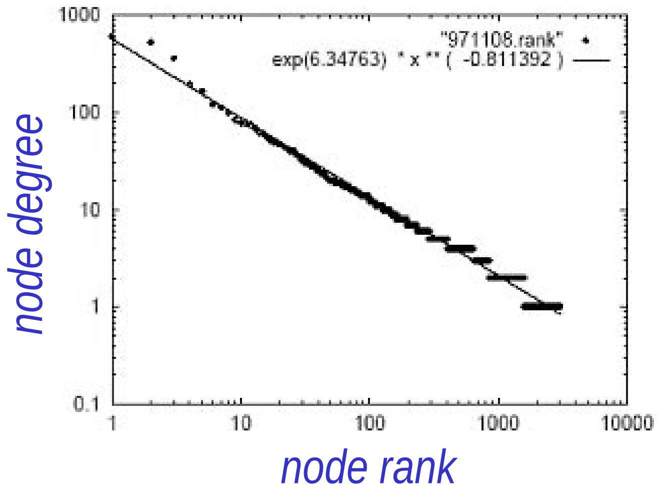


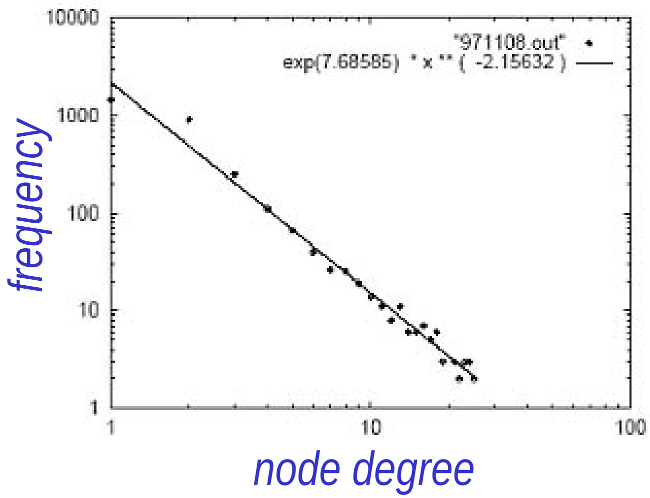
Map of Internet colored by IP address

(Bill Cheswick & Hal Burch, http://research.lumeta.com/ches/map)

2. A Complex Systems Sampler

d. Complex networks – *Case studies: Internet*





Two power laws of the Internet topology

(Faloutsos, Faloutsos & Faloutsos, 1999)

- the connectivity degree of a node follows a power of its rank (sorting out in decreasing order of degree):

  node degree ~ (node rank)$^{-\alpha}$

- the most connected nodes are the least frequent:

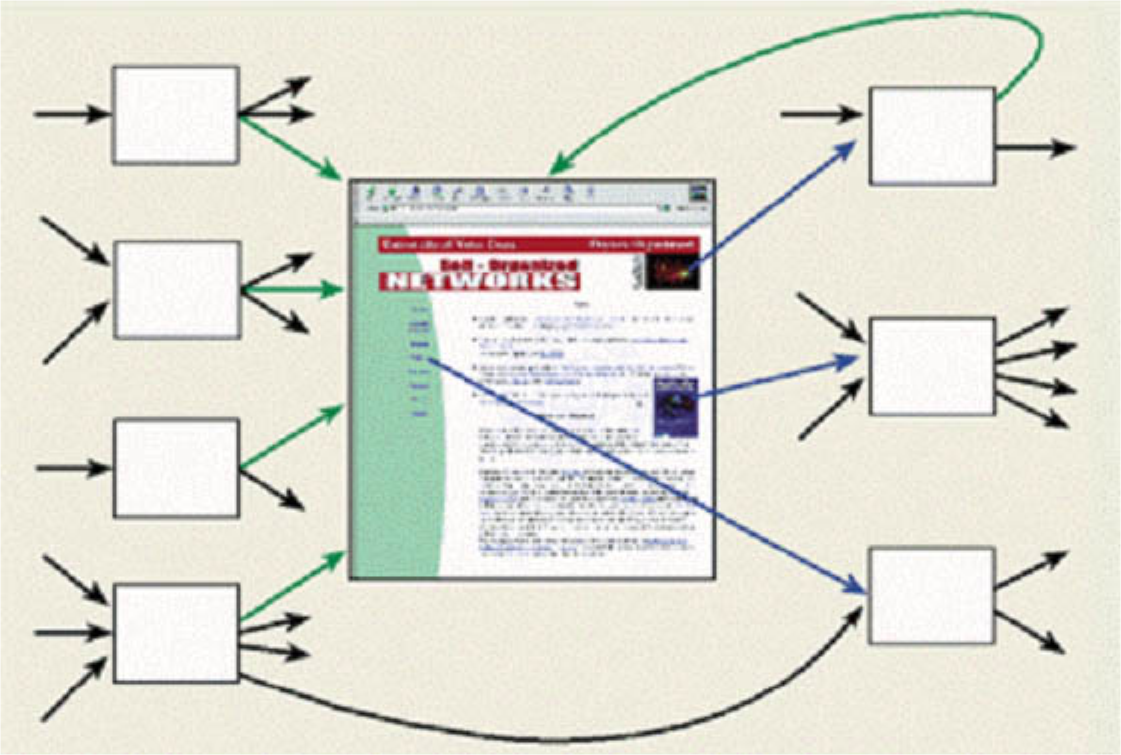
  degree frequency ~(node degree)$^{-\gamma}$

  $$P(k) \sim k^{-\gamma}$$

$\rightarrow$ *the Internet is a scale-free network*

2. A Complex Systems Sampler

d. Complex networks – *Case studies: World Wide Web*

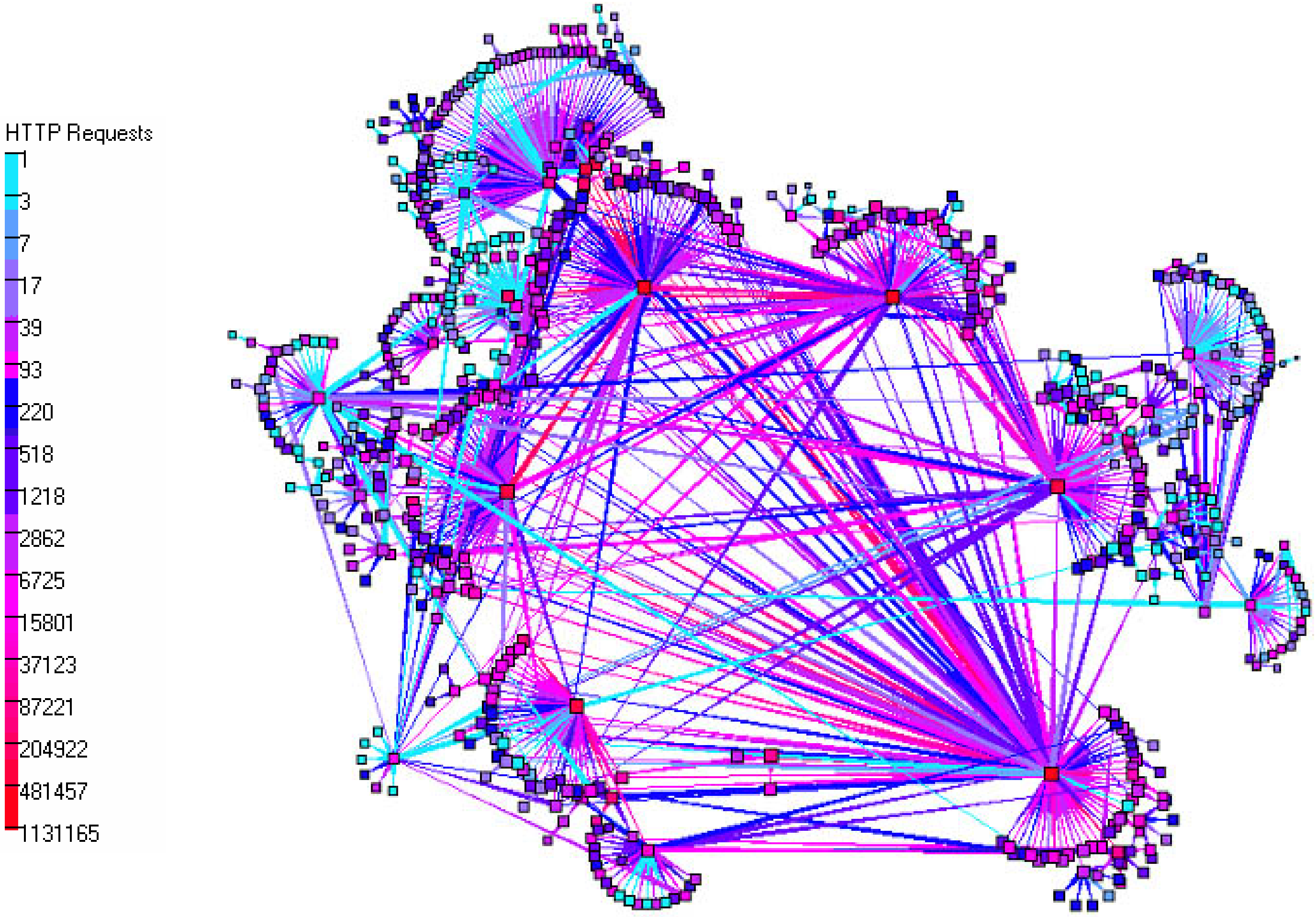


Schema of the World Wide Web of documents

- the World Wide Web is a network of documents that reference each other
- the nodes are the Web pages and the edges are the hyperlinks
- edges are directed: they can be outgoing and incoming hyperlinks

2. A Complex Systems Sampler

d. Complex networks – *Case studies: World Wide Web*

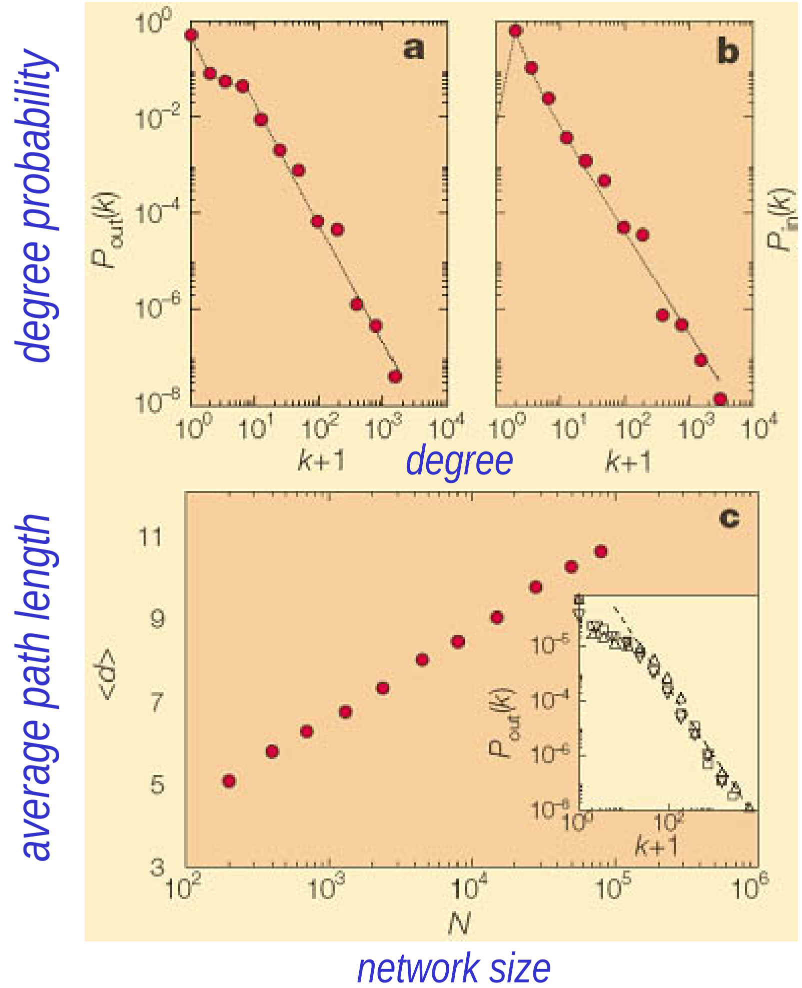


Hierachical topology of the international Web cache

(Bradley Huffaker, http://www.caida.org/tools/visualization/plankton)

2. A Complex Systems Sampler

d. Complex networks – *Case studies: World Wide Web*



Distribution of links on the World-Wide Web

(Albert, Jeong & Barabási, 1999)

- WWW is a scale-free network:

$$P(k) \sim k^{-\gamma}$$

with $\gamma_{out} = 2.45$ and $\gamma_{in} = 2.1$

- WWW is also a small world:

$$L \approx \alpha \ln N$$

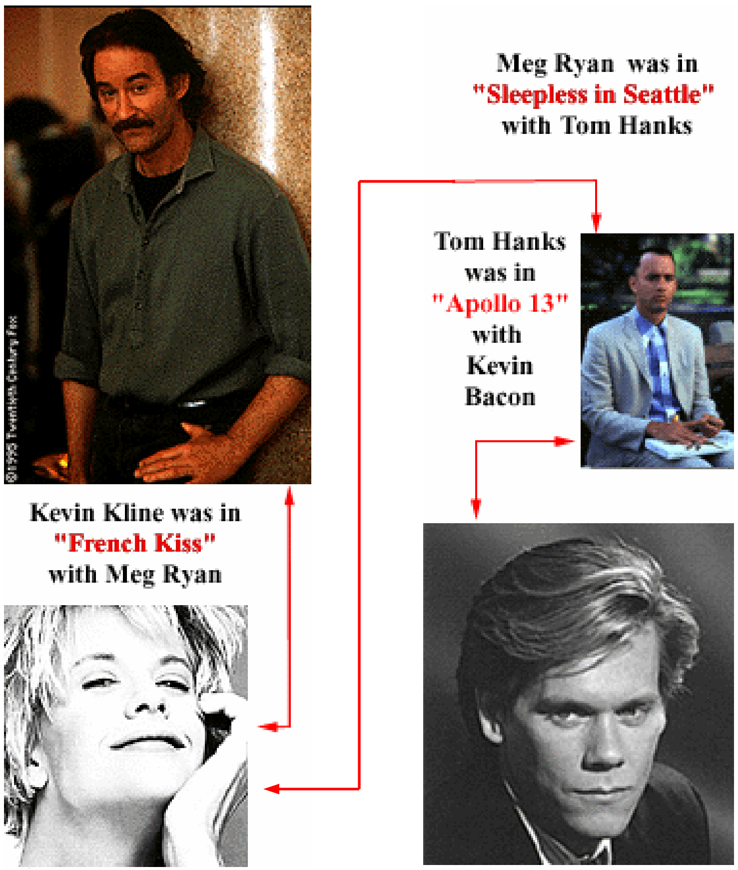
with $L \approx 11$ for $N = 10^5$ documents

2. A Complex Systems Sampler

d. Complex networks – *Case studies: actors*

"The Oracle of Bacon"

http://www.cs.virginia.edu/oracle



Path from K. Kline to K. Bacon = 3 (as of 1995)

(http://collegian.ksu.edu/issues/v100/FA/n069/fea-making-bacon-fuqua.html)

- a given actor is on average 3 movies away from Kevin Bacon (L_{Bacon}=2.946, as of June 2004) . . . or any other actor for that matter

- Hollywood is a small world

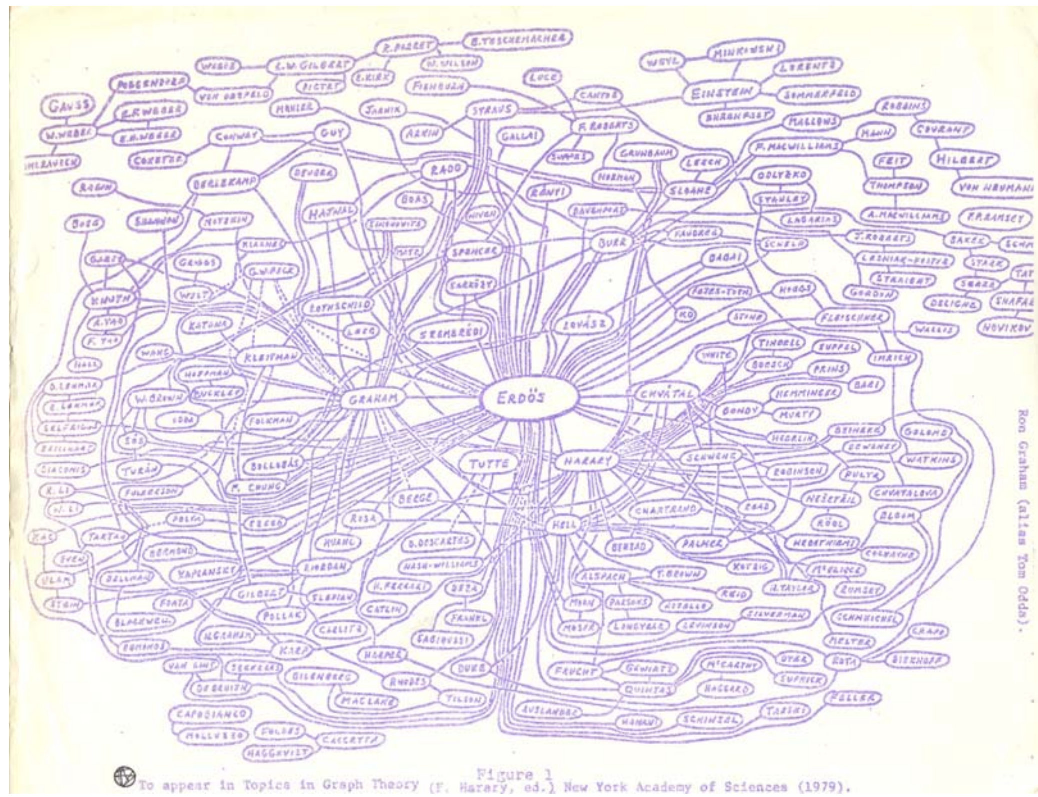
- . . . and it is a *scale-free* small world: a few actors played in a lot of movies, and a lot of actors in few movies

2. A Complex Systems Sampler

d. Complex networks – *Case studies: scientists*

"The Erdős Number Project"

http://www.oakland.edu/enp



Co-authors of Paul Erdős have number 1, co-authors of co-authors number 2, etc.

Mathematicians form a highly clustered (C = 0.14) small world (L = 7.64)

Complex Systems Made Simple

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2. A Complex Systems Sampler

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   b. Pattern formation
   c. Swarm intelligence
   d. Complex networks
   e. Spatial communities:
      - ***Spatial ecology***
      - ***Evolutionary games***
   f. Structured morphogenesis

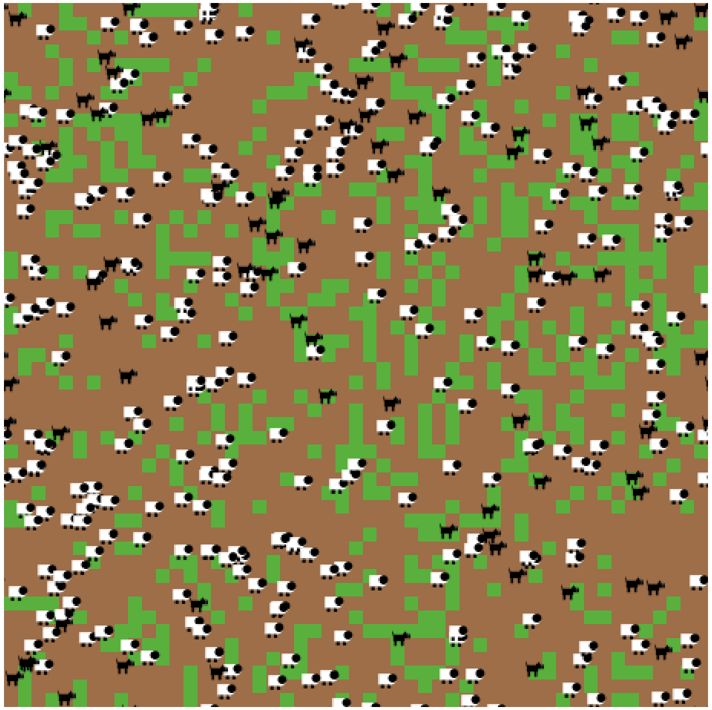
3. Commonalities

4. NetLogo Tutorial

2. A Complex Systems Sampler

e. Spatial communities – *Spatial ecology*

NetLogo model: /Biology/Wolf Sheep Predation

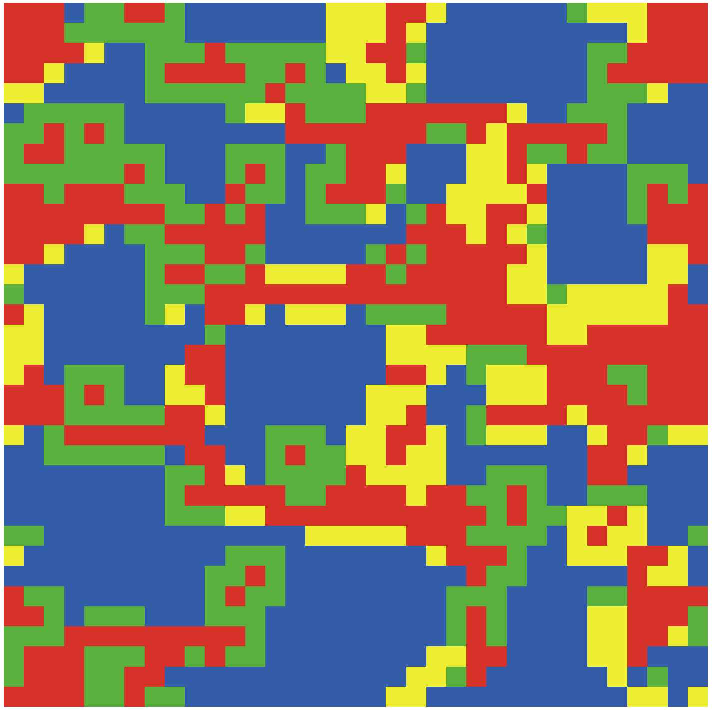


➢ *explore model on your own!*

2. A Complex Systems Sampler

e. Spatial communities – *Evolutionary games*

NetLogo model: /Social Science/Unverified/Prisoner's Dilemma/PD Basic Evolutionary



- *explore model on your own!*

Complex Systems Made Simple

1. Introduction
2. A Complex Systems Sampler
   a. Cellular automata
   b. Pattern formation
   c. Swarm intelligence
   d. Complex networks
   e. Spatial communities
   f. **Structured morphogenesis**
3. Commonalities
4. NetLogo Tutorial

2. A Complex Systems Sampler — f. Morphogenesis

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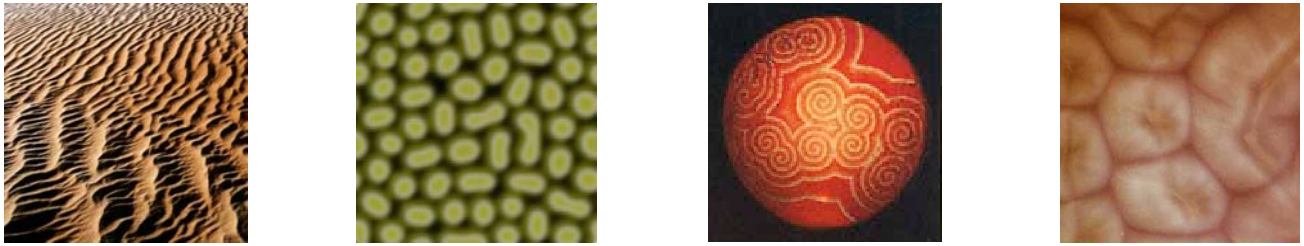
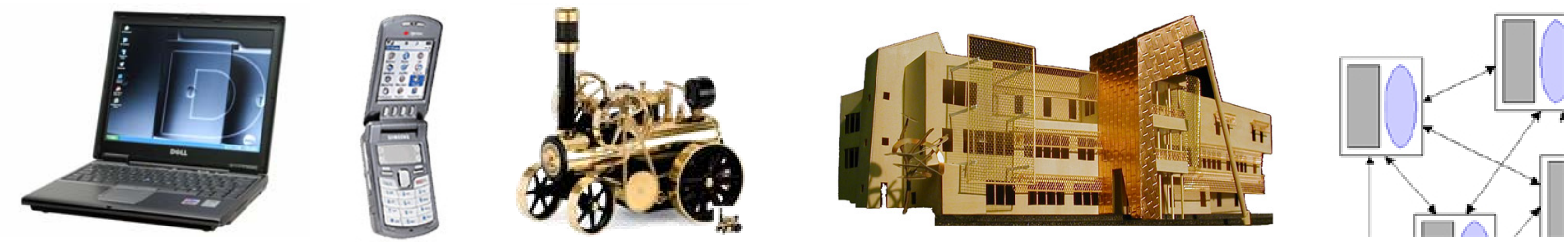
- **Non-bio/social self-organization exhibits "simple" patterns**
  - ✓ physical/chemical systems: ripples (sand dunes), spots, stripes & waves (reaction-diffusion), convection cells (hot fluids), etc.



  - ✓ huge diversity of substrates and scales, yet mostly like textures: repetitive, statistically **uniform**, "information-poor"
  - ✓ spontaneous order from amplification of **random** fluctuations
  - ✓ **unpredictable** number and position of meso entities (spots, stripes)
  - → *the only natural systems able to create complex, reproducible structures are biological and social*

2. A Complex Systems Sampler — f. Morphogenesis

- Non-bio/social structures are deliberately designed
  - ✓ engineered systems: electronics, vehicles, buildings, software, etc.



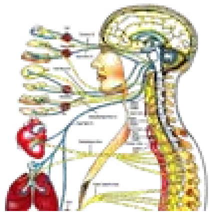
  - ✓ human constructions are **complicated** and **reproducible**
  - ✓ diversity of parts and **modules**, arranged in specific ways
  - ✓ fundamentally **heterogeneous** and “information-rich”
  - ✓ however, necessity of centralized, exogenous design & planning

→ *the only structured forms that are also spontaneously emergent are biological and social*

2. A Complex Systems Sampler — f. Morphogenesis

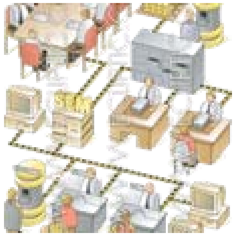
- Biological organisms are self-organized <u>&</u> structured

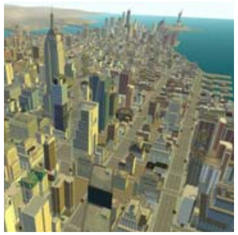


  - ✓ mesoscopic organs and limbs have intricate, **nonrandom** morphologies
  - ✓ development is highly **reproducible** in number and position of body parts
  - ✓ **heterogeneous** elements arise under information-rich genetic control

- Techno-social collectives are self-organized <u>&</u> structured



  - ✓ termite mounds, companies, networks, cities, etc.
  - ✓ although less tightly arranged than organisms, social structures are also heterogenous and modular, mixing hierarchy and heterarchy

2. A Complex Systems Sampler — f. Morphogenesis

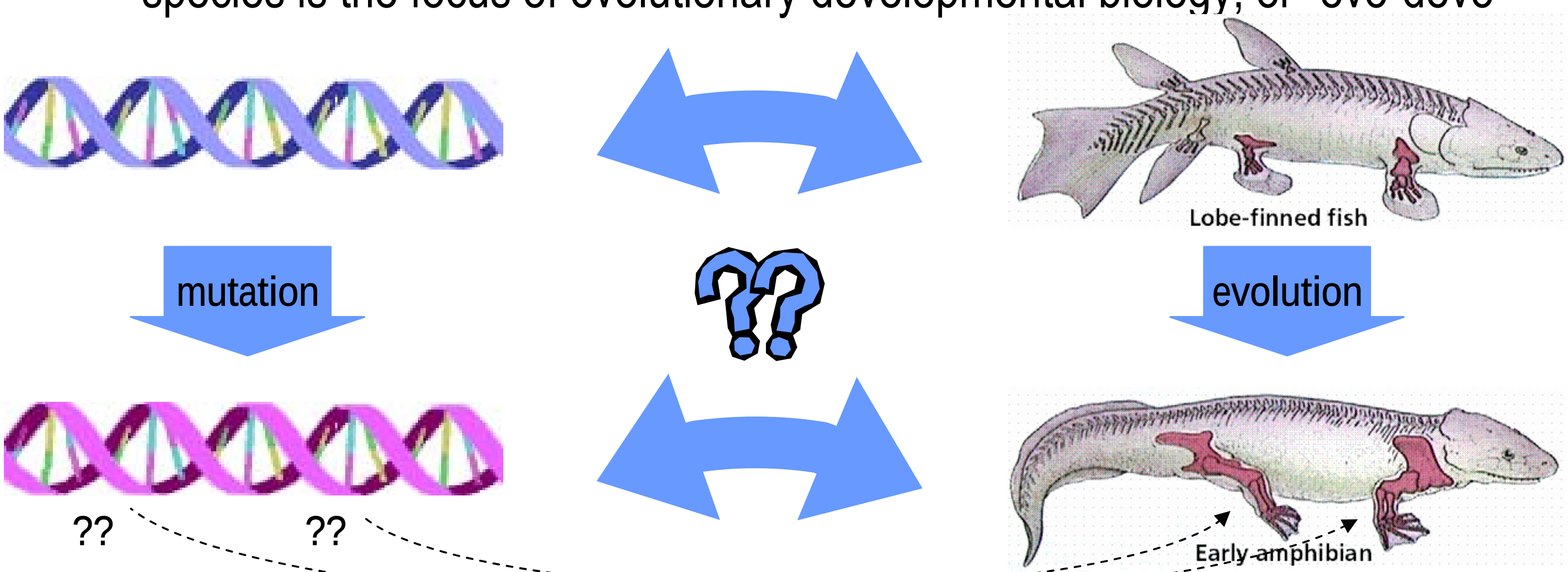
- Self-organized complex structures rely on ***informed*** and ***positioned*** agents
  - ✓ the unique feature of biological and social morphogenesis is that *agents (cells, insects, computers, humans) carry* ***sophisticated instruction sets*** *(DNA, behavioral rules, program, cognition)* and possess a minimal "awareness" of their location within the system
    - functional information vastly superior to inert units of matter
    - source of nontrivial behavioral repertoire, creating agent diversity by *position-dependent* differentiation, and by evolution
    - allows rich agent combinations, recombinations, i.e., hierarchical constructions based on reusable modules

→ *how does "genotypical" control at the agent level lead to complex "phenotypical" self-organization?*

2. A Complex Systems Sampler — f. Morphogenesis

➢ Development: the missing link of the Modern Synthesis

- ✓ biology's "Modern Synthesis" demonstrated the *existence* of a fundamental correlation between genotype and phenotype, yet the molecular and cellular *mechanisms* responsible for development are still unclear
- ✓ the genotype-phenotype link cannot remain an abstraction if we want to unravel the generative laws of development and evolution
- ✓ understanding variation by comparing the actual development of different species is the focus of evolutionary developmental biology, or "evo-devo"



Purves et al., *Life: The Science of Biology*

2. A Complex Systems Sampler — f. Morphogenesis

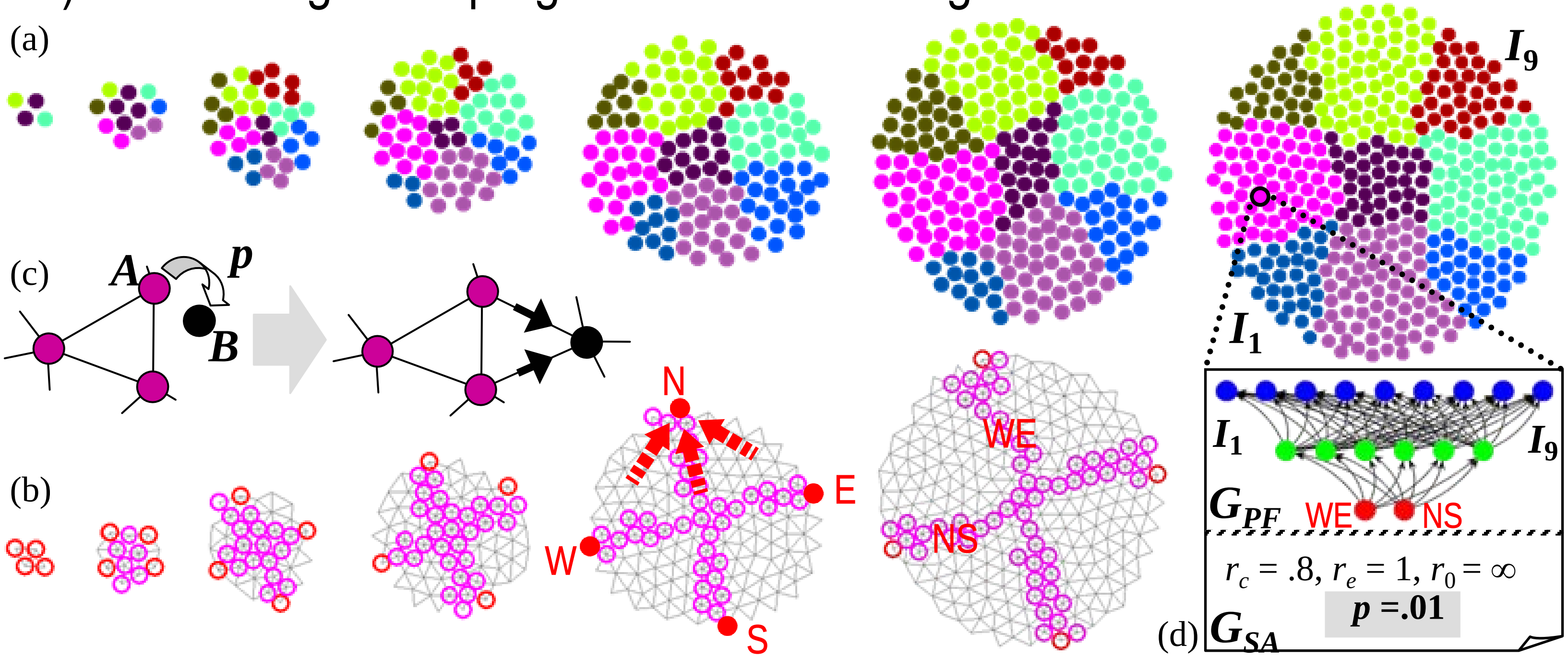
➢ Simultaneous growth and patterning (SA + PF)

a) swarm growing from 4 to 400 agents by division

b) swarm mesh, gradient midlines; pattern is continually maintained by source migration, e.g., *N* moves away from *S* and toward *WE*

c) agent *B* created by *A*'s division quickly submits to SA forces and PF traffic

d) combined genetic programs inside each agent



2. A Complex Systems Sampler — f. Morphogenesis

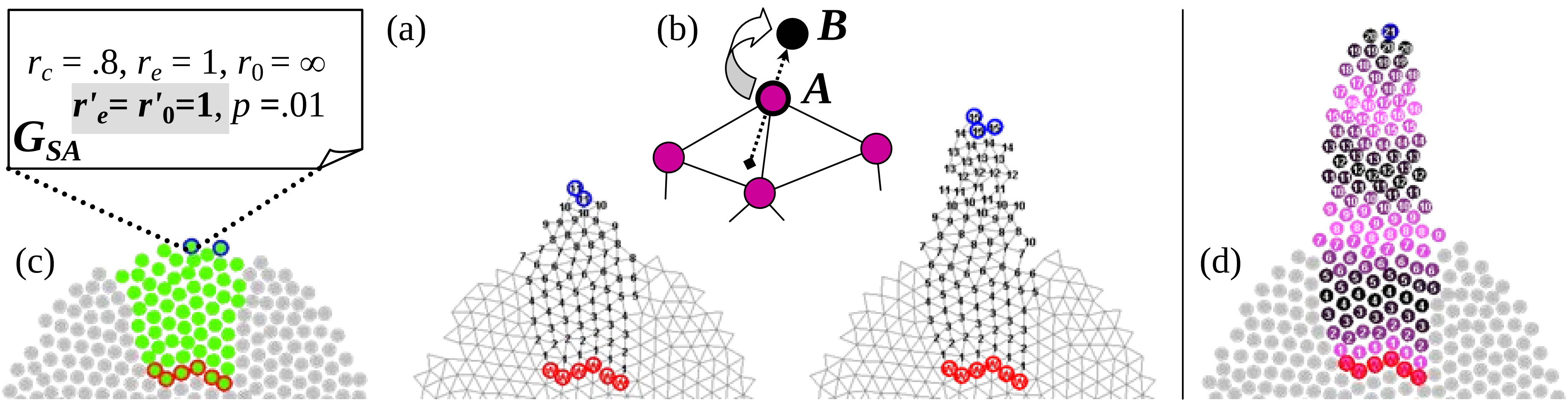
➢ Modular, anisotropic growth (SA[k])

a) genetic SA parameters are augmented with repelling V values r'_e and r'_0 used between the growing region (green) and the rest of the swarm (gray)

b) daughter agents are positioned away from the neighbors' center of mass

c) offshoot growth proceeds from an "apical meristem" made of gradient ends (blue circles)

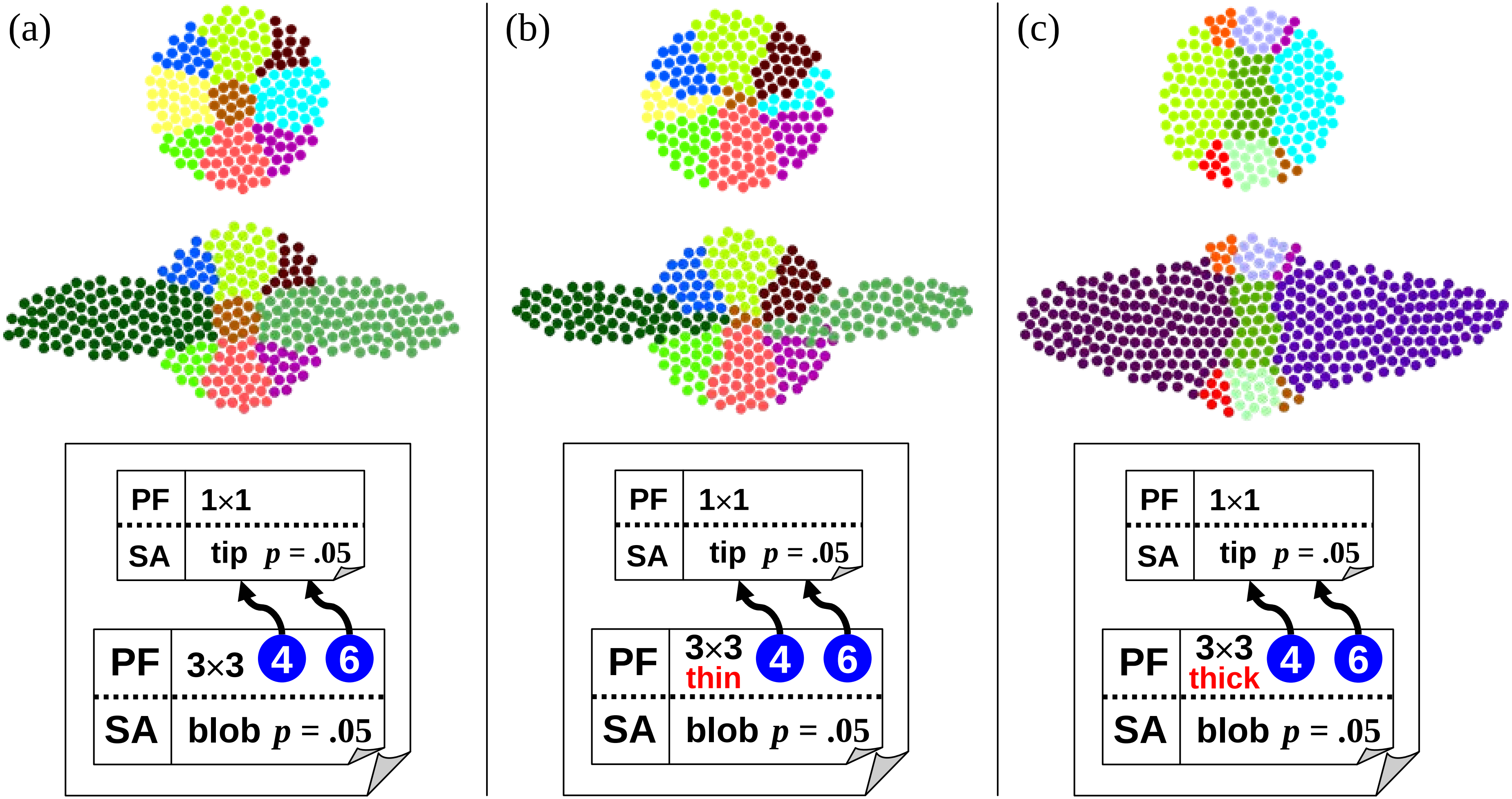
d) the gradient underlying this growth



2. A Complex Systems Sampler — f. Morphogenesis

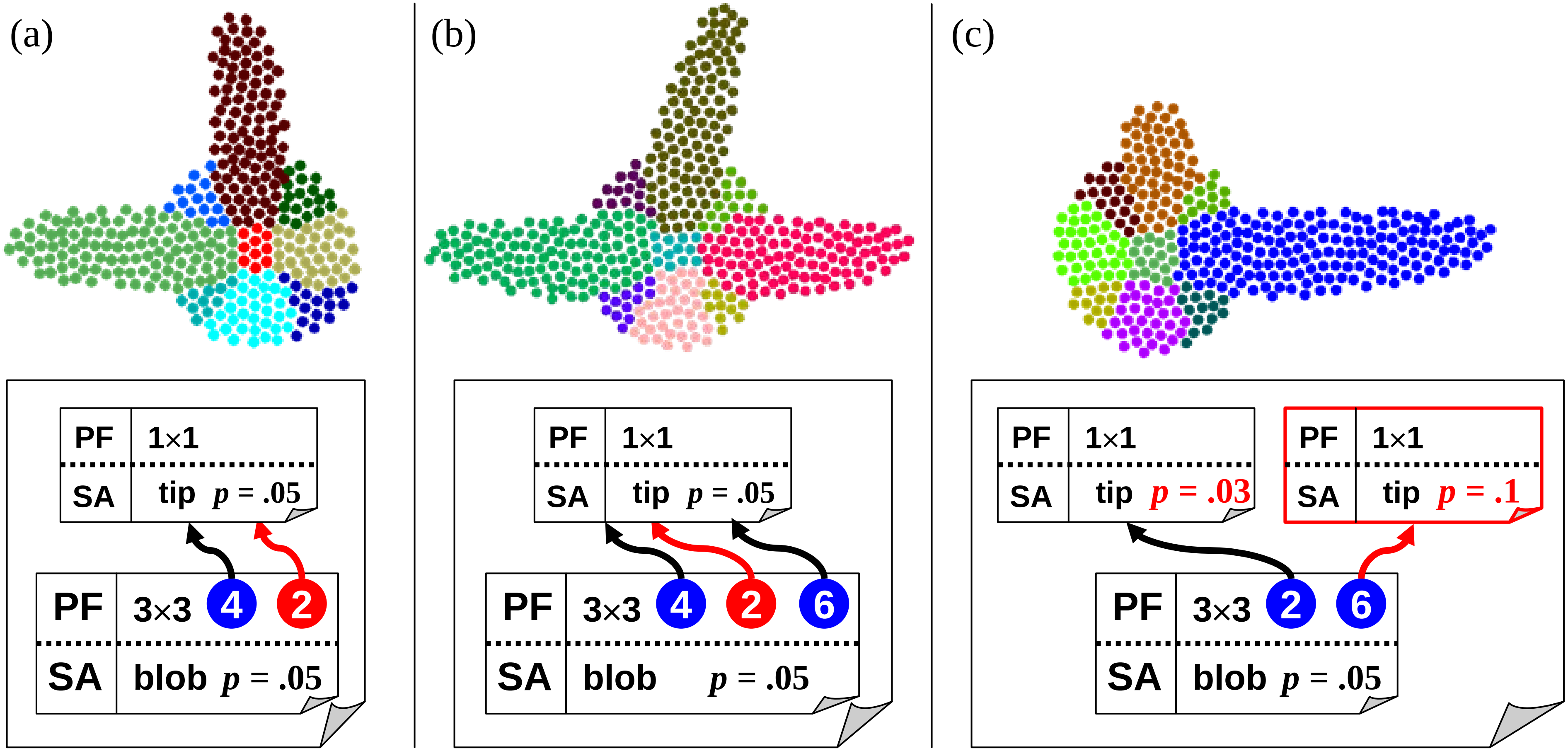
➢ Modular growth and patterning (SA[k] + PF[k]): 2 levels

a) wild type; b) "thin" mutation of the base body plan; c) "thick" mutation



2. A Complex Systems Sampler — f. Morphogenesis

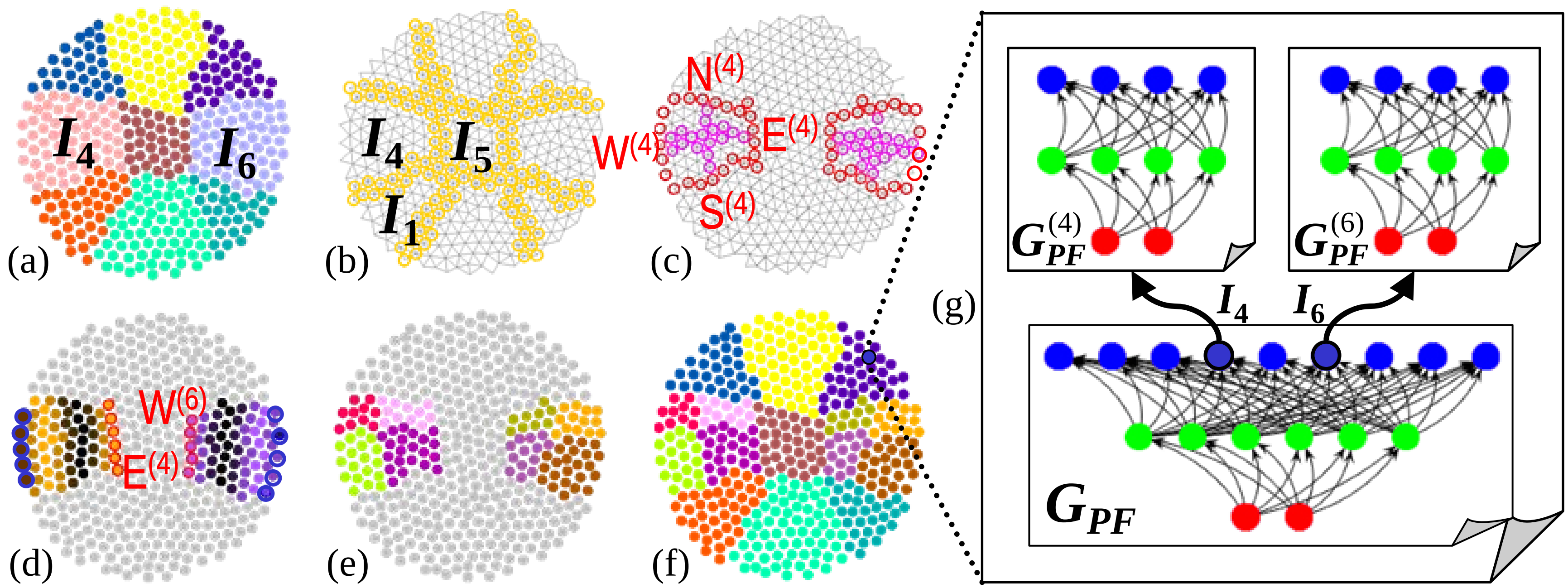
- Modular growth and patterning (SA[k] + PF[k]): 2 levels
  - a) antennapedia; b) homology by *duplication*; c) *divergence* of the homology



2. A Complex Systems Sampler — f. Morphogenesis

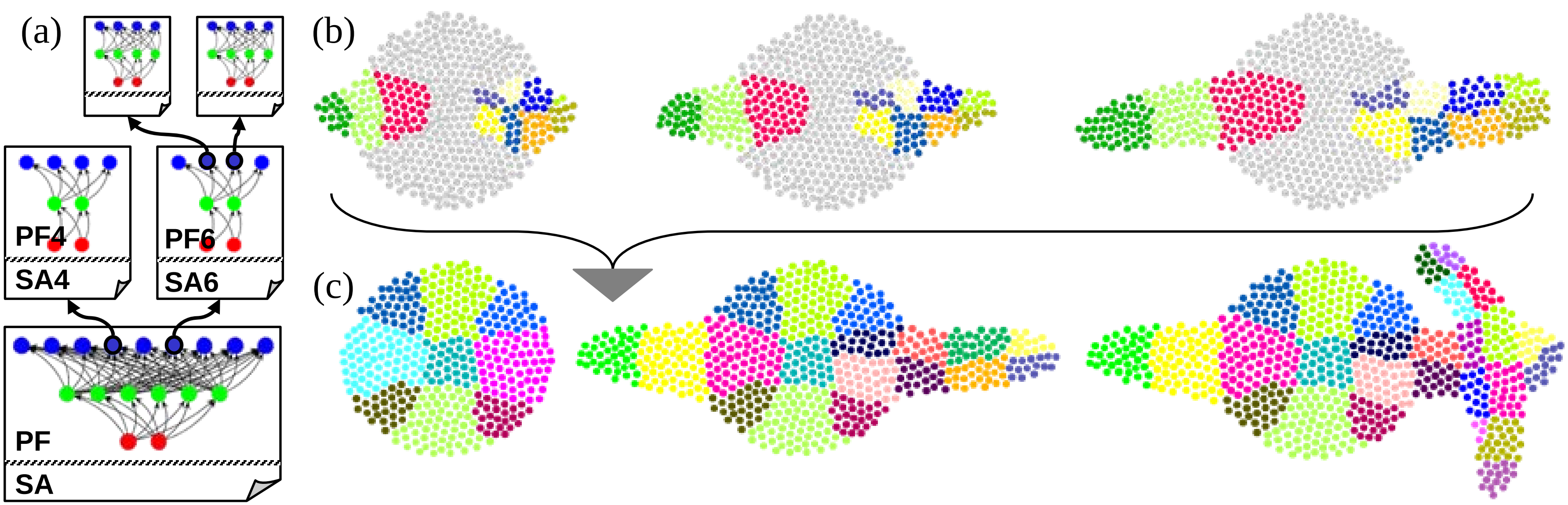
➢ Modular, recursive patterning (PF[k])

- b) border agents highlighted in yellow
- c) border agents become new gradient sources inside certain identity regions
- d) missing border sources arise from the ends (blue circles) of other gradients
- e) & f) subpatterning of the swarm in I_4 and I_6
- g) corresponding hierarchical gene regulation network



2. A Complex Systems Sampler — f. Morphogenesis

- Modular growth and patterning (SA[k] + PF[k]): 3 levels
  - a) example of a three-level modular genotype giving rise to the artificial organism on the right
  - b) three iterations detailing the simultaneous limb-like growth process and patterning of these limbs during execution of level 2 (modules 4 and 6)
  - c) main stages of the complex morphogenesis, showing full patterns after execution of levels 1, 2 and 3.



2. A Complex Systems Sampler — f. Morphogenesis

➢ Modular growth and patterning (SA[k] + PF[k]): 3 levels