Coding Positional Information with Phases: Pattern Recognition by Wave-Matching

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Pattern Recognition by Wave-Matching

1. Active Perception
2. Graph Matching
3. Phase Tagging
4. Dynamic Temporal Matching
5. Lattice Wave Induction

Pattern Recognition by Wave-Matching

1. Active Perception
   - Ascribing structure to data
   - Gestalt: bottom-up self-organization
   - Schemas: top-down guided organization
   - Perceptual and cognitive schemas
2. Graph Matching
3. Phase Tagging
4. Dynamic Temporal Matching
5. Lattice Wave Induction

1. Active Perception

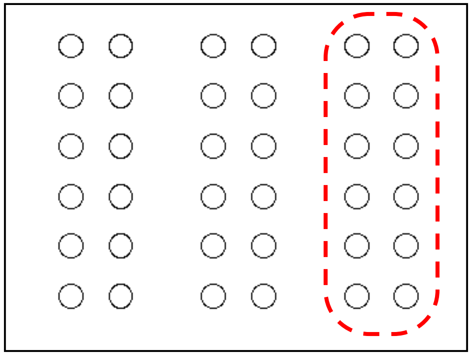
➢ Ascribing structure to data

- ✓ active perception means actively *organizing* raw sensory data
- ✓ feature grouping & segmentation proceed on two levels
  - *bottom-up* self-organization of low-level features → Gestalt principles
  - *top-down* pattern recognition → pre-recorded schemas

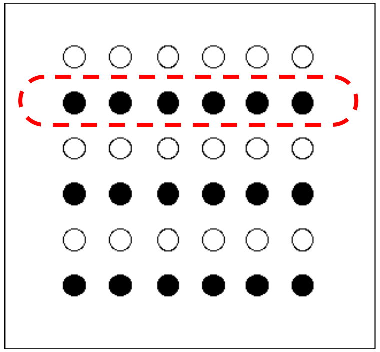
1. Active Perception

➢ Gestalt: bottom-up self-organization

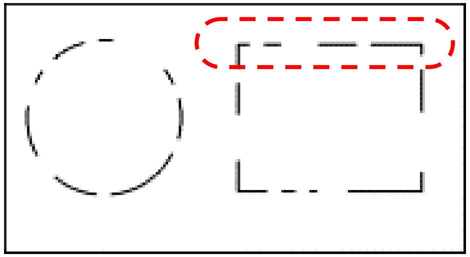
- ✓ low-level organizational principles group features *locally*
- ✓ proximity



- ✓ similarity



- ✓ closure/continuity

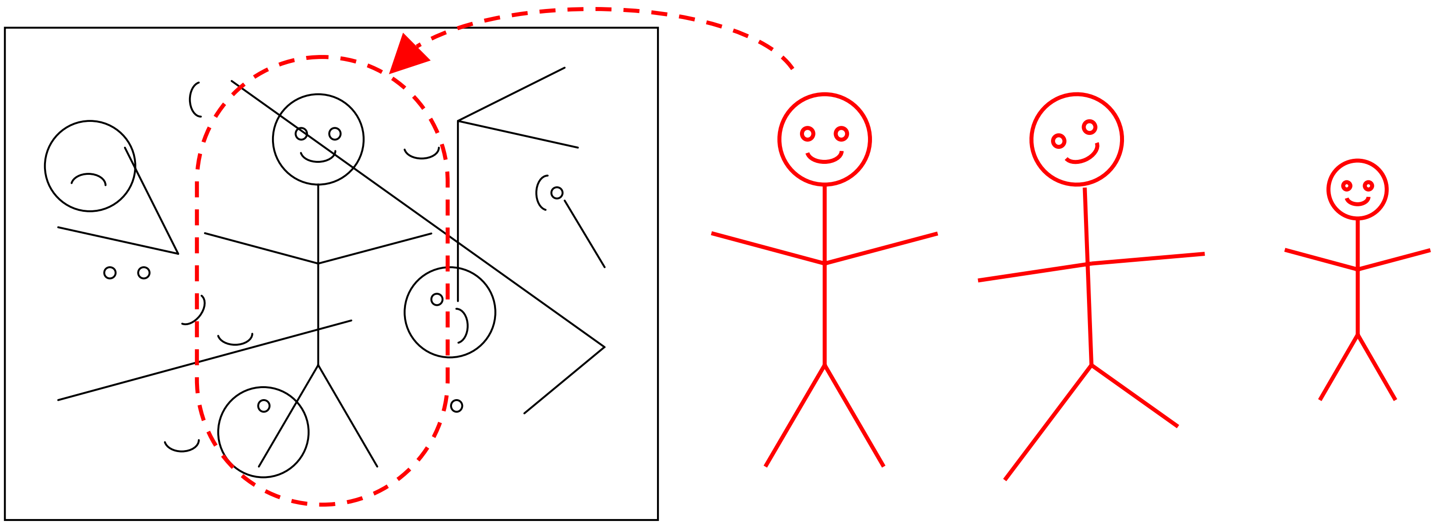


- ✓ symmetry
- ✓ common fate
- ✓ etc.

1. Active Perception

➢ Schemas: top-down guided organization

- ✓ high-level stored patterns finish grouping features *globally*
- ✓ local regularities or statistical properties are not enough: recognition must be guided by *schemas*
  - schemas are *constrained*: specific assemblage of components
  - yet also *flexible*: invariant by rotation, translation, scaling, distortions

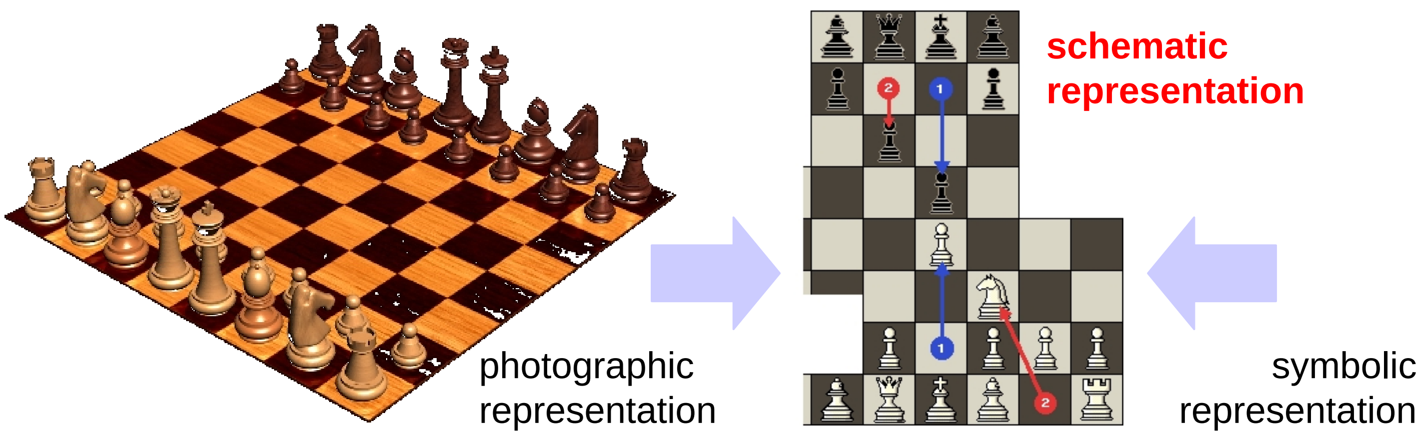


1. Active Perception

➢ Perceptual and cognitive schemas

- ✓ schemas (“mental representations”) are simplified but representative models of cognitive categories
- ✓ they contain no details but have an overall resemblance with their object, mixing analogic and symbolic information
- ✓ ex: “mental imagery”, “geons”, “cognitive linguistic icons”, etc.

→ ***compositionality***: *components, modules, building blocks*



1.e4 c5 2.Nf3 e6 3.d4 cxd4 4.Nxd4 Nf6 5.Nc3 d6 6.g4 Be7 7.g5 Nfd7 8.Rg1 Nc6 9.Be3 Nb6 10.Qd2 Bd7 11.0-0-0 Nxd4 12.Bxd4 0-0 13.f4 Rc8 14.h4 Nc4 15.Bxc4 Rxc4 16.Qd3 b5 17.f5 e5 18.Be3 Re8 19.h5 Qa5 20.g6 hxg6 21.hxg6 Bc6 22.Bh6 Bf8 23.gxf7+ Kxf7 24.Qxd6 Kg8 25.Rxg7+ 1-0

Pattern Recognition by Wave-Matching

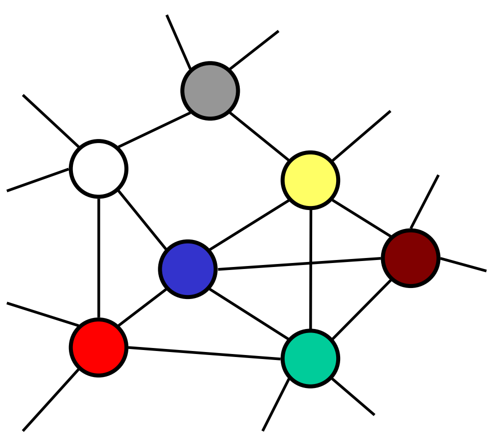
1. Active Perception
2. Graph Matching
   - Schemas as graph templates
   - Top-down schema application as graph matching
   - Elastic graph matching
3. Phase Tagging
4. Dynamic Temporal Matching
5. Lattice Wave Induction

2. Graph Matching

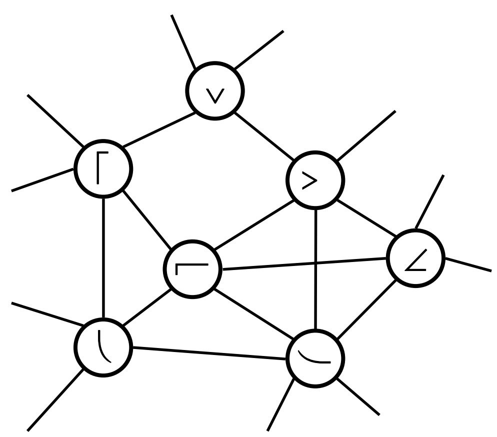
➢ Schemas as graph templates

- ✓ *graphs* provide a general relational format of representation especially appropriate for modeling schemas

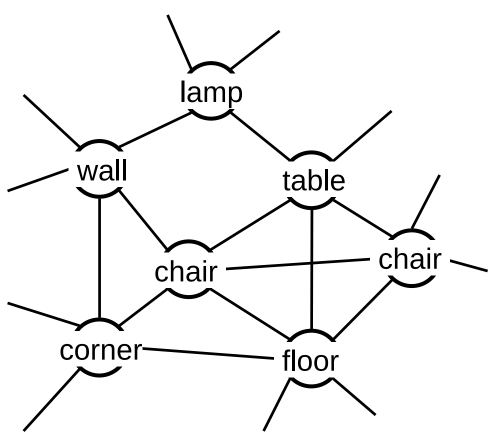
- ✓ graphs are “constellations of features”, in which
  - nodes carry ***labels*** $\rightarrow$ symbolic information
  - links carry *geometrical relationships* $\rightarrow$ analogic information



pixels



contours



symbols/objects

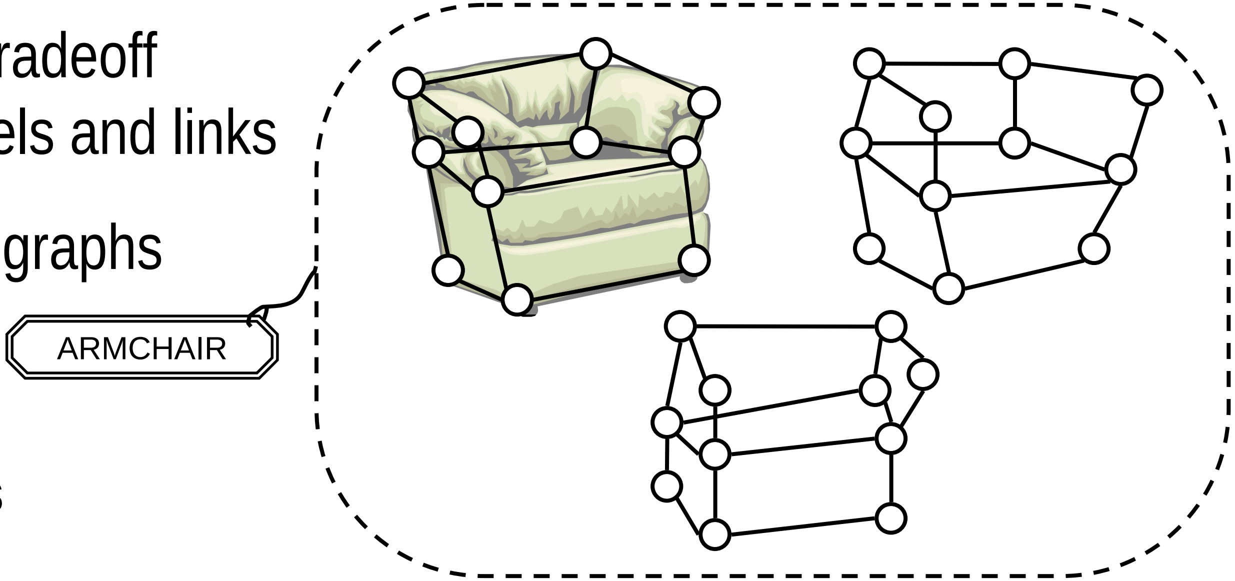
2. Graph Matching

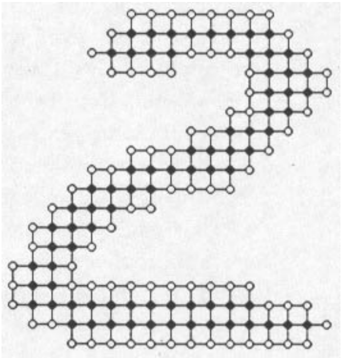
➢ Schemas as graph templates

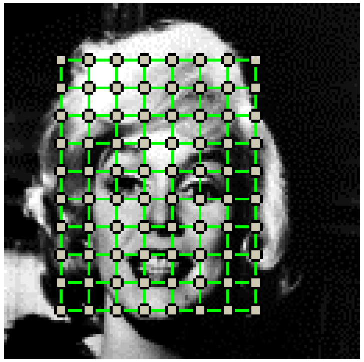
✓ information tradeoff between labels and links

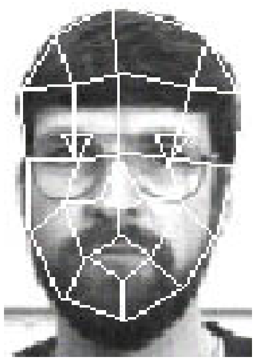
✓ examples of graphs

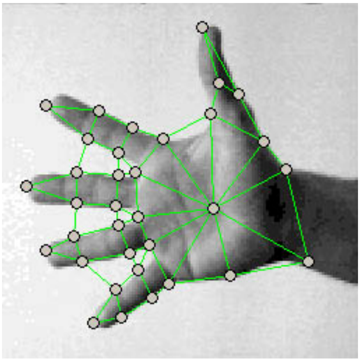
- objects
- faces
- characters
- etc.











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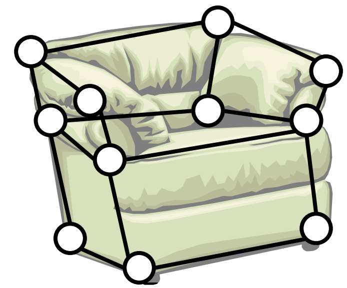
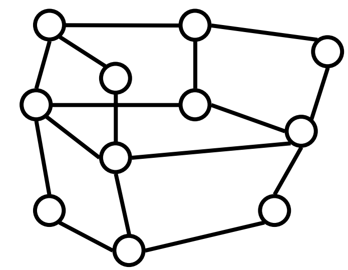
2. Graph Matching

➢ Top-down schema application as graph matching

✓ expectation: graphs representing the same object category are structurally similar

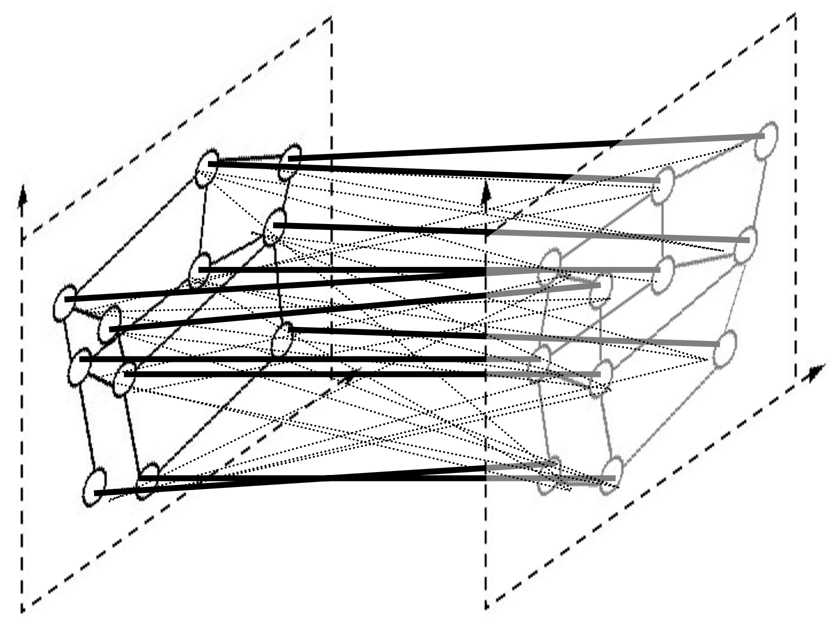




→ *modeling schemas as **deformable templates***

✓ graph templates can be directly compared by ***graph matching***

→ *establishment of a dynamical link mapping*

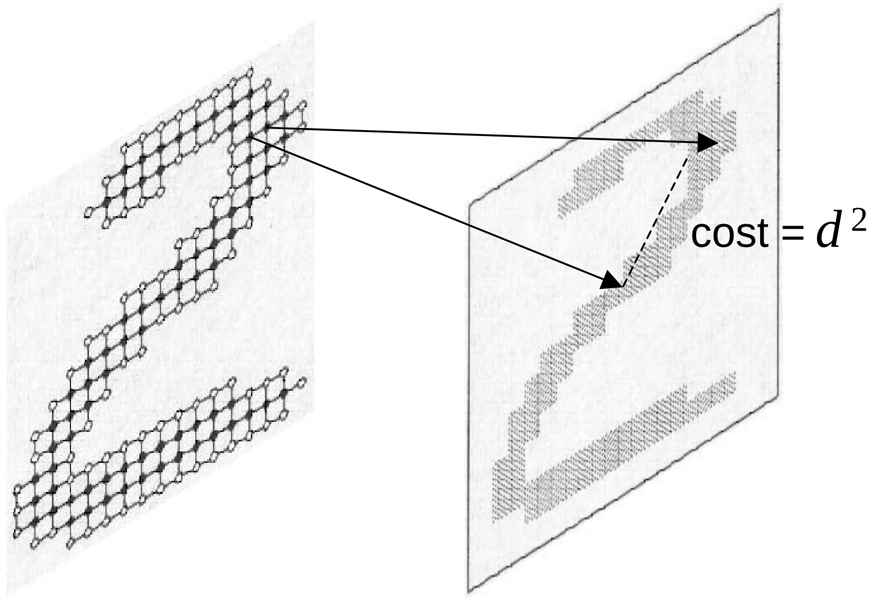


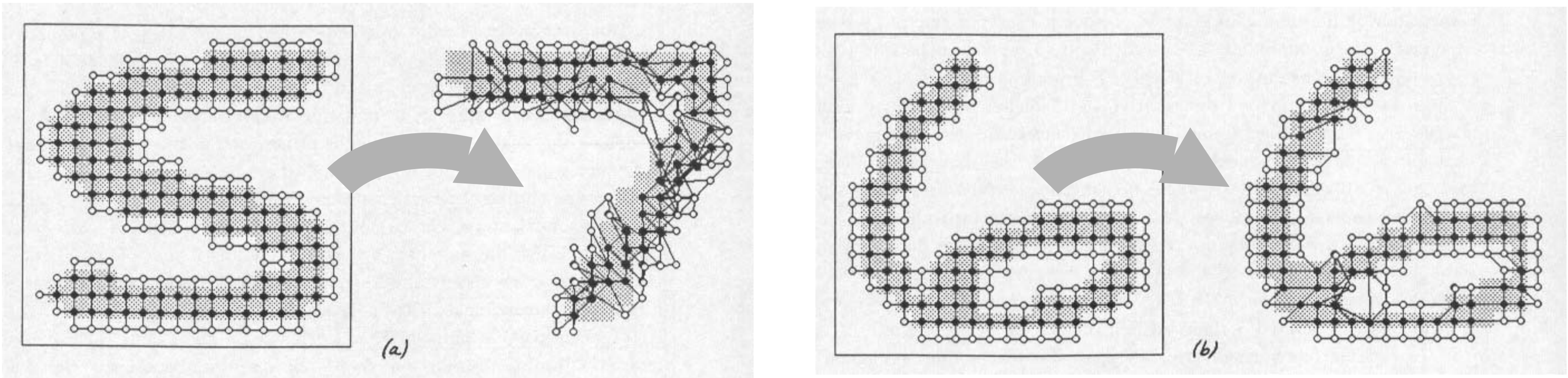
2. Graph Matching

➢ Elastic graph matching

- one link per node
- minimize distance
- minimize label difference

→ *link mapping equivalent to an* ***elastic deformation***





Bienenstock and Doursat (1994) *A shape-recognition model using dynamical links.*

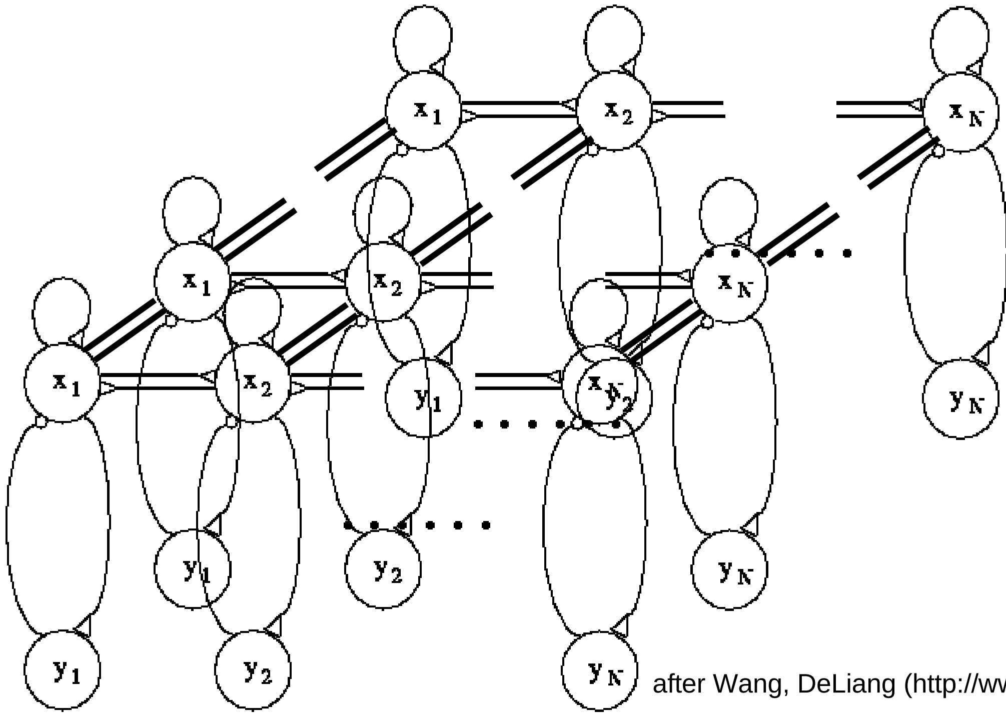
Pattern Recognition by Wave-Matching

1. Active Perception
2. Graph Matching
3. Phase Tagging
   - Temporal coding of graphs
   - Coupled oscillatory units
   - Block synchronization: segmentation
   - Traveling waves: positional information
4. Dynamic Temporal Matching
5. Lattice Wave Induction

3. Phase Tagging

➢ Temporal coding of graphs

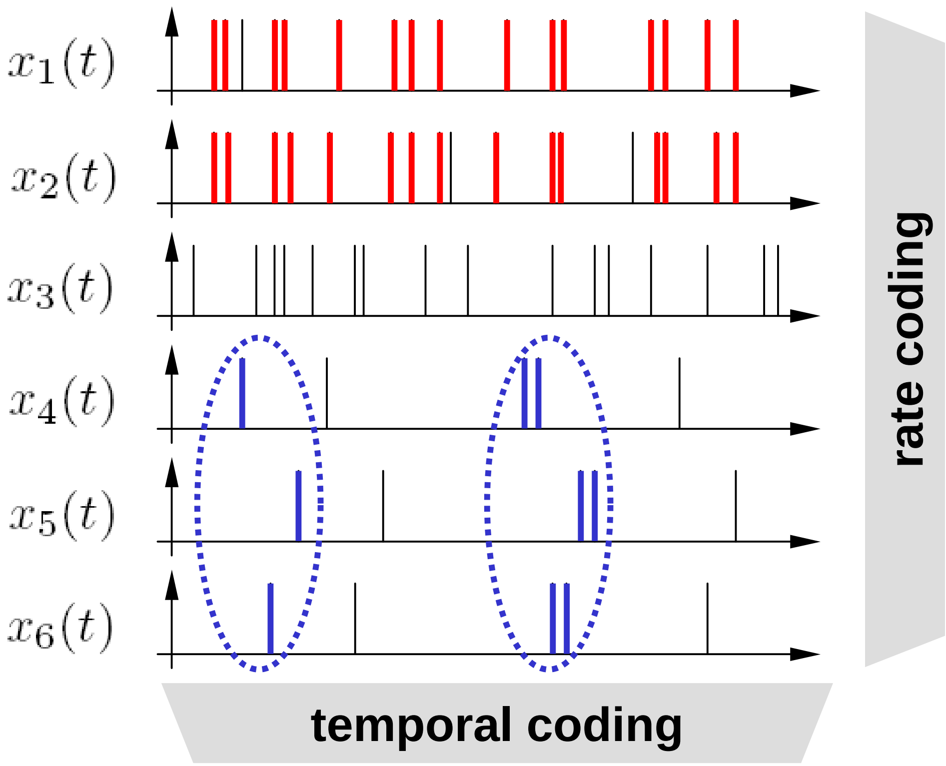
- ✓ main idea: a schema is a graph, where a graph is a network of *coupled temporal units* — spiking, excitable, oscillatory, etc.
  - nodes = timings (phases)
  - links = timing (phase) differences



after Wang, DeLiang (http://www.cse.ohio-state.edu/~dwang/)

3. Phase Tagging

➢ Temporal coding of graphs



$\langle x_1(t) \rangle =$ ● high activity rate

$\langle x_2(t) \rangle =$ ● high activity rate

$\langle x_3(t) \rangle =$ ● high activity rate

$\langle x_4(t) \rangle =$ ○ low activity rate

$\langle x_5(t) \rangle =$ ○ low activity rate

$\langle x_6(t) \rangle =$ ○ low activity rate

$$\langle x_1(t)\, x_2(t) \rangle \gg \langle x_1(t)\, x_3(t) \rangle$$

➢ 1 and 2 more in sync than 1 and 3

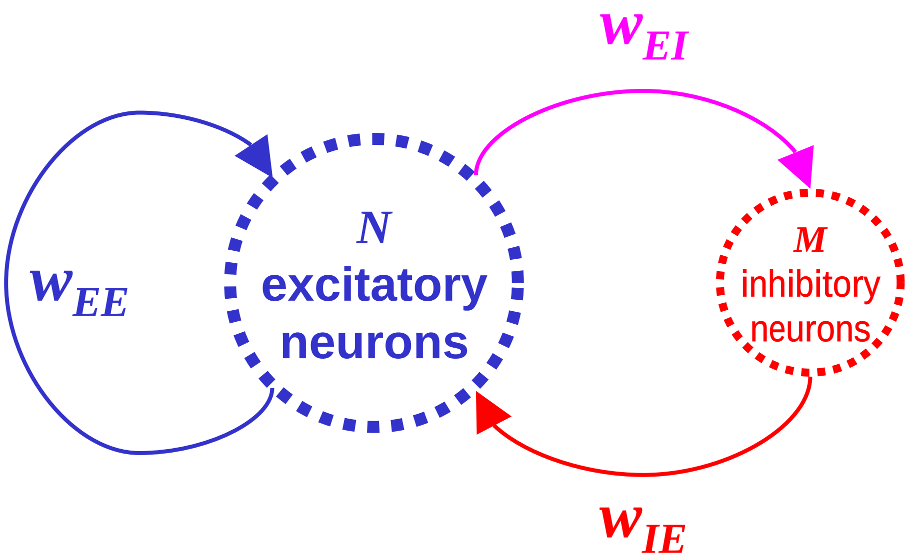
$$\langle x_4(t)\, x_5(t-\tau_{4,5})\, x_6(t-\tau_{4,6}) \rangle$$

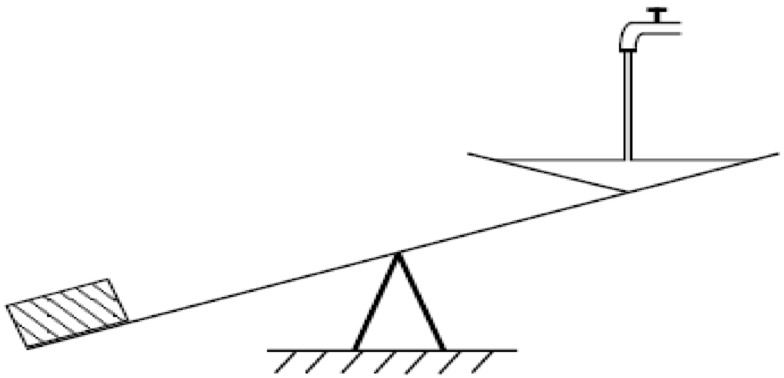
➢ 4, 5 and 6 correlated through delays

3. Phase Tagging

➢ Coupled oscillatory units

- ✓ example: dual excitatory-inhibitory system
- ✓ this system is a *relaxation oscillator*, i.e., exhibits discontinuous jumps
- ✓ different from sinusoidal or harmonic oscillations



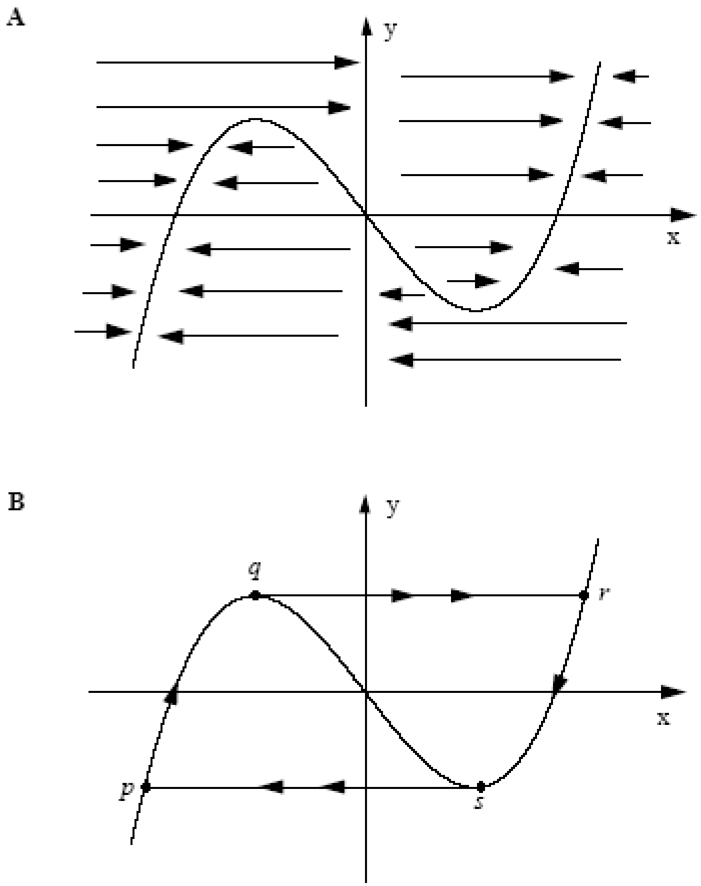


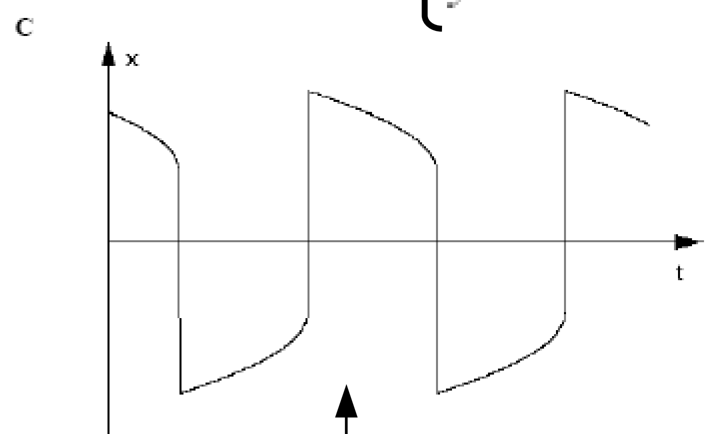
after Wang, DeLiang (http://www.cse.ohio-state.edu/~dwang/)

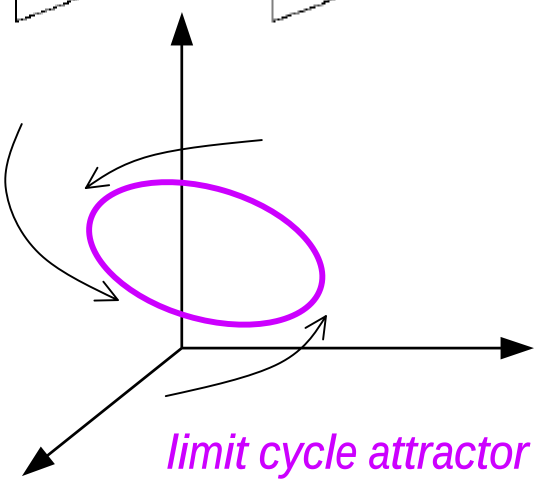
3. Phase Tagging

➢ Coupled oscillatory units

✓ Van der Pol oscillator $\ddot{x} + x = c(1 - x^2)\dot{x} \Leftrightarrow \begin{cases} \dot{x} = c(y - f(x)) \\ \dot{y} = -x / c \end{cases}$







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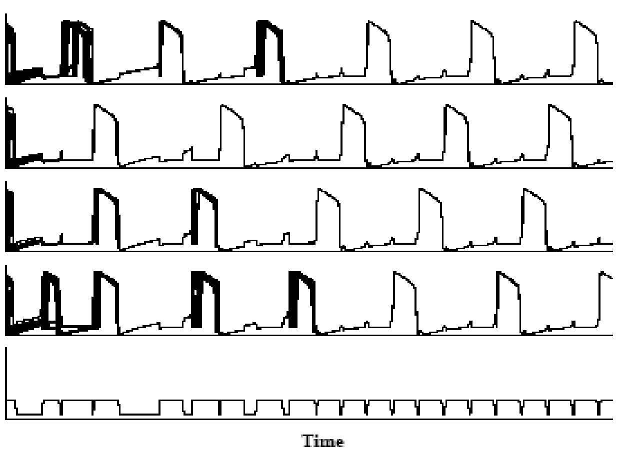
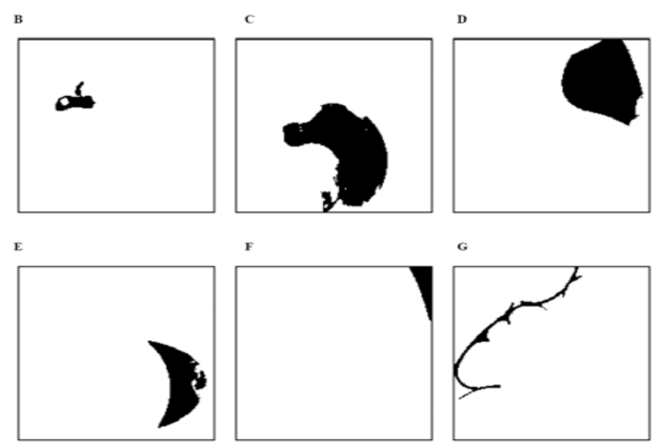
3. Phase Tagging

➢ Block synchronization: segmentation

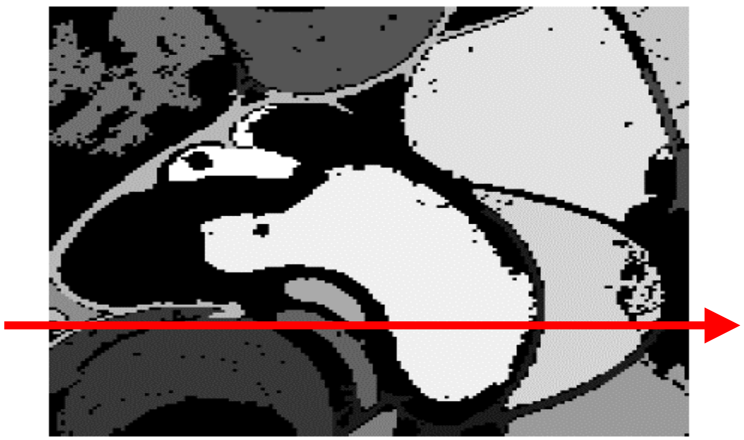
- ✓ a model of segmentation by sync: LEGION (Wang & Tierman)

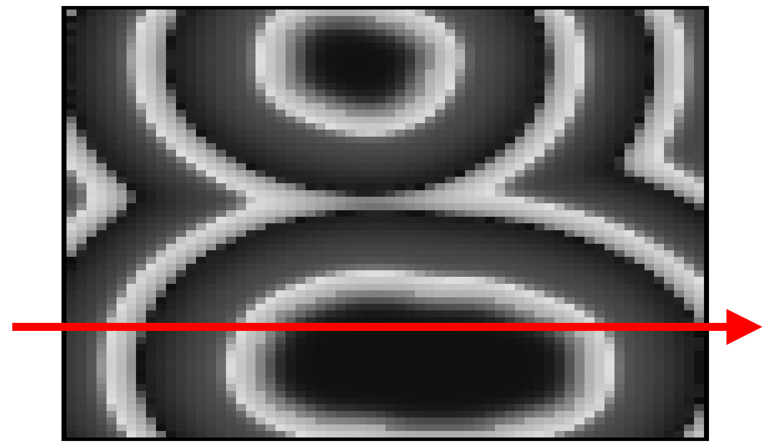


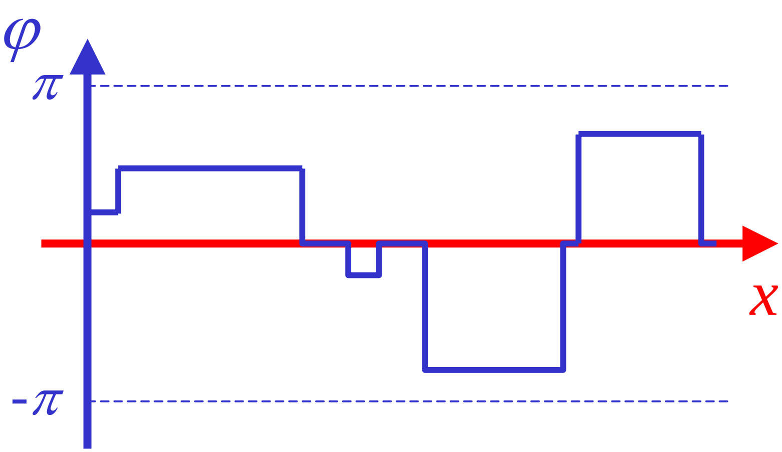


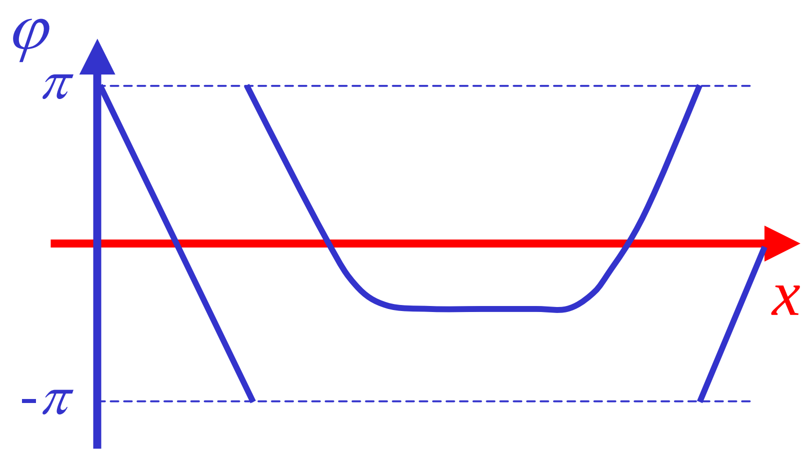
3. Phase Tagging

- Traveling waves: positional information
  - ✓ instead of phase plateaus → *phase gradients*









Pattern Recognition by Wave-Matching

1. Active Perception
2. Graph Matching
3. Phase Tagging
4. Dynamic Temporal Matching
   - Excitable units & delayed coupling
   - Onset of spatiotemporal patterns (STPs)
   - Phases as coordinates
   - 1-D and 2-D dynamic phase matching
5. Lattice Wave Induction

4. Dynamic Temporal Matching

➢ Excitable units

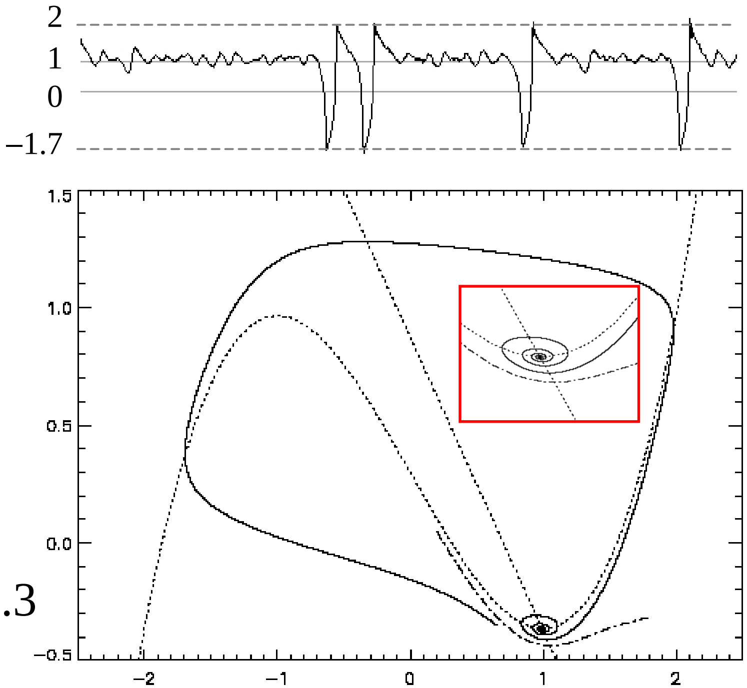
✓ a Bonhoeffer-van der Pol (BvP) oscillator has two main regimes:

a) sparse, stochastic → *excitable*

b) quasi-periodic → *oscillatory*

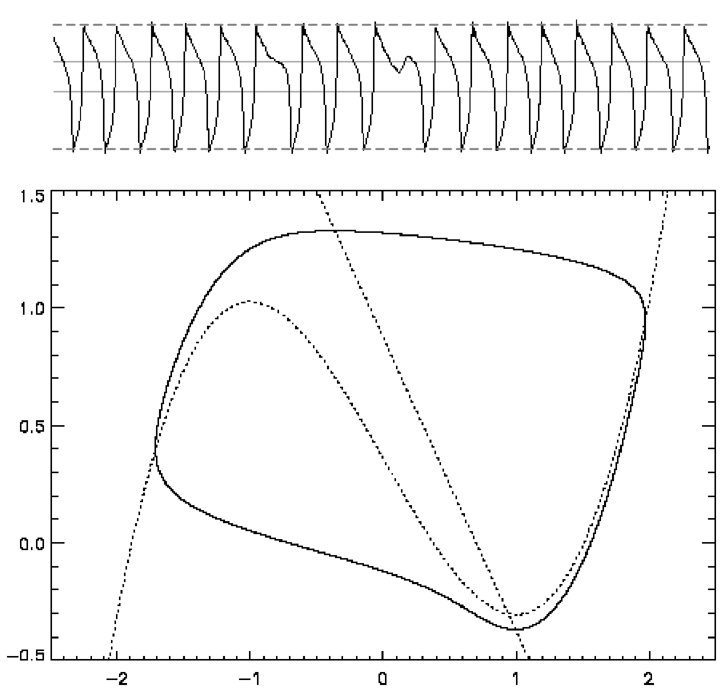
$$\begin{cases} \dfrac{du_i}{dt} = c\left(u_i - \dfrac{u_i^3}{3} + v_i + z\right) + \eta \\ \dfrac{dv_i}{dt} = \dfrac{1}{c}\left(a - u_i - bv_i\right) + \eta \end{cases}$$

(a)



$z = -.3$

(b)



$z = -.36$

4. Dynamic Temporal Matching

➢ Delayed coupling

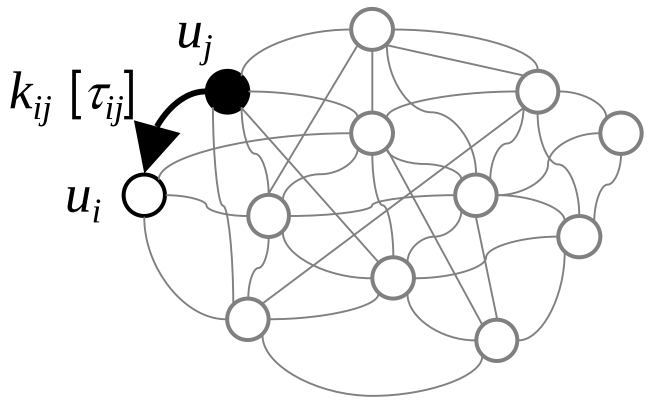
- ✓ fully connected net of BvP units

$$\begin{cases} \dfrac{du_i}{dt} = c\left(u_i - \dfrac{u_i^3}{3} + v_i + z\right) + \eta + K_i \\ \dfrac{dv_i}{dt} = \dfrac{1}{c}\left(a - u_i - bv_i\right) + \eta \end{cases}$$

$$K_i(t) = \sum_{\substack{j=1 \\ u_j(t-\tau_{ij})<0}}^{N} k_{ij}\Big(u_j(t-\tau_{ij}) - u_i(t)\Big)$$

- ✓ $i \leftarrow j$ coupling features:
  - proportional to u-signal difference (only in spiking domain $u < 0$)
  - positive connection weight k_{ij}
  - nonzero transmission delay τ_{ij}
- ✓ delays verify a rule of transitivity:
  - $\tau_{ij} + \tau_{jk} \cong \tau_{ik} \pmod{T}$
- ✓ . . .equivalent to per-node times:

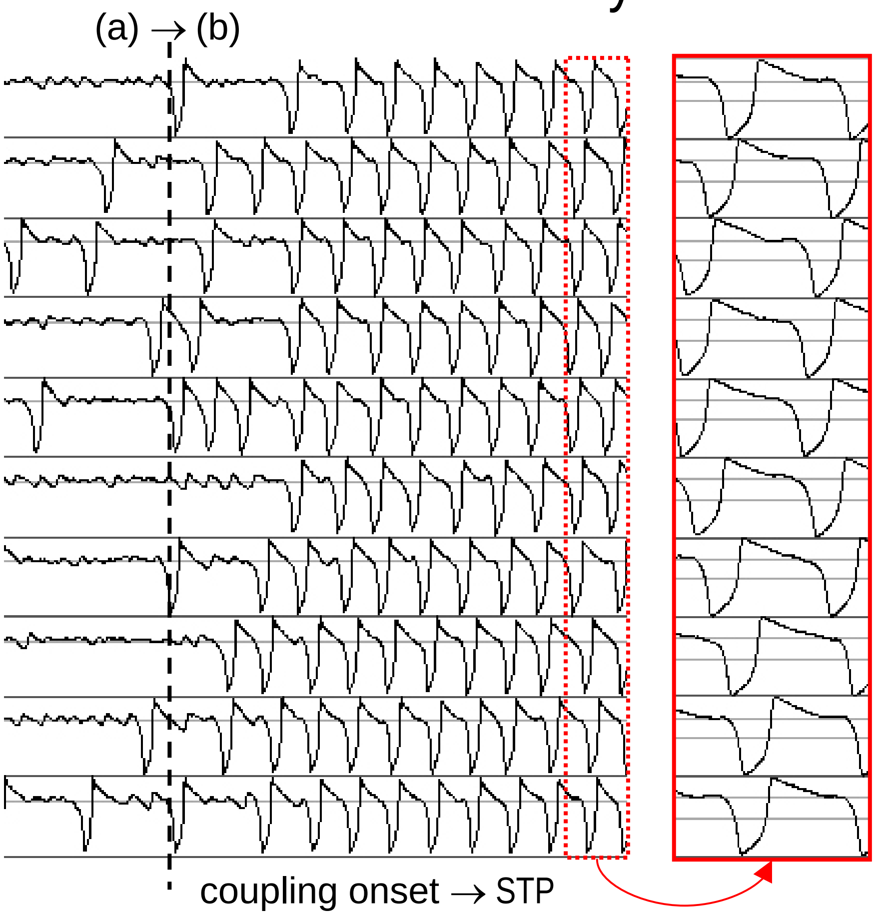
$$\Leftrightarrow \quad \tau_{ij} = (\theta_i - \theta_j) \bmod T$$



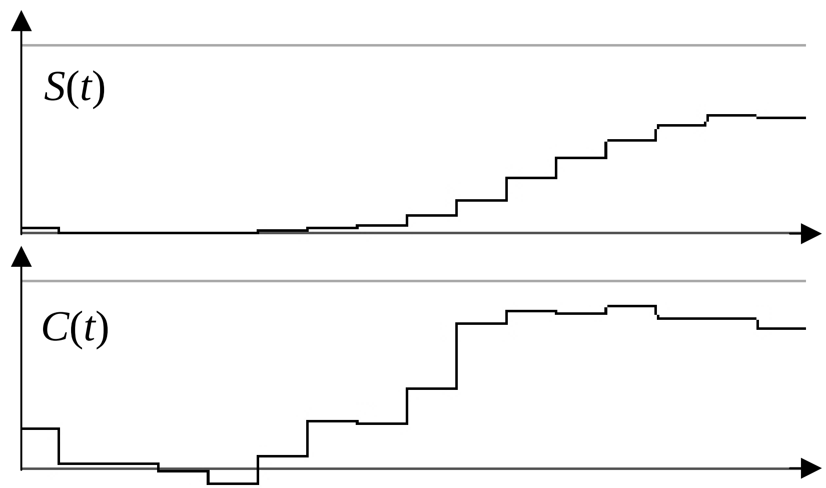
4. Dynamic Temporal Matching

➢ Onset of spatiotemporal patterns (STPs)

- ✓ when coupling is turned on, units transition from regime (a) to (b) and exhibit delayed correlations $t_i - t_j$ in accordance with τ_{ij}



$$S(t) = \frac{1}{N(N-1)} \sum_{i \neq j} \langle u_i(t')\ u_j(t' - \tau_{ij}) \rangle_{t-T_s}^{t}$$

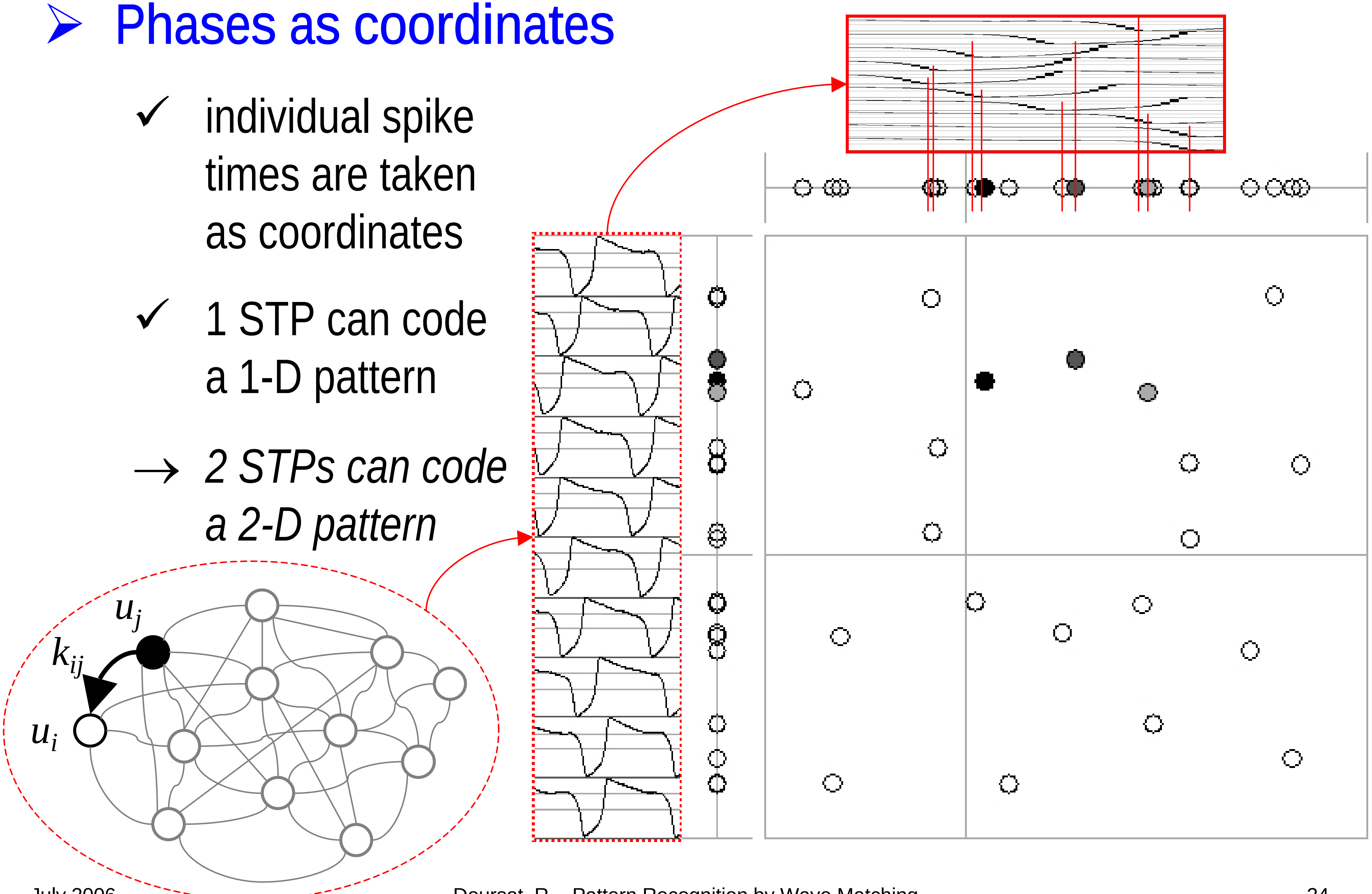


$$C(t) = \frac{1}{N(N-1)} \sum_{i \neq j} \cos\left(\frac{2\pi}{T}(t_i(t) - t_j(t) - \tau_{ij})\right)$$

4. Dynamic Temporal Matching

➢ Phases as coordinates

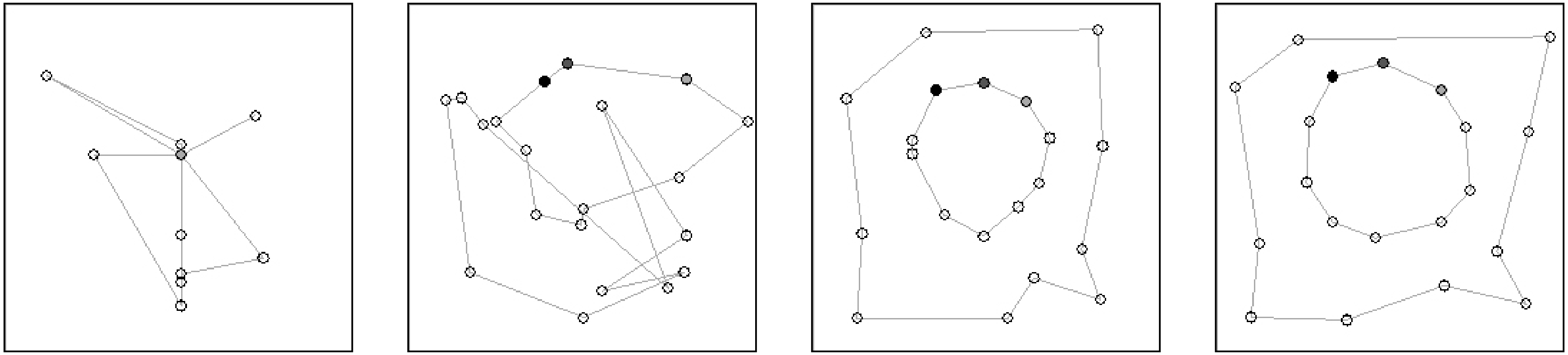
- ✓ individual spike times are taken as coordinates
- ✓ 1 STP can code a 1-D pattern
- → *2 STPs can code a 2-D pattern*



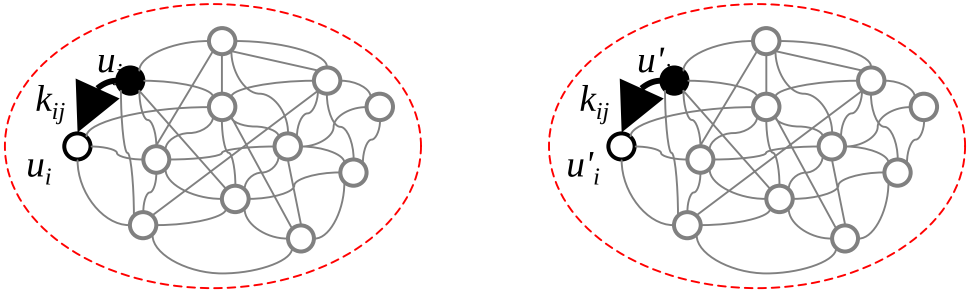
4. Dynamic Temporal Matching

➢ Phases as coordinates

- ✓ the simultaneous onset of a pair of STPs is graphically equivalent to the unfolding of a 2-D constellation of dots



- ✓ note: the two STPs can be on two different networks or they can alternate on the same network (see waves on a lattice in 5.)



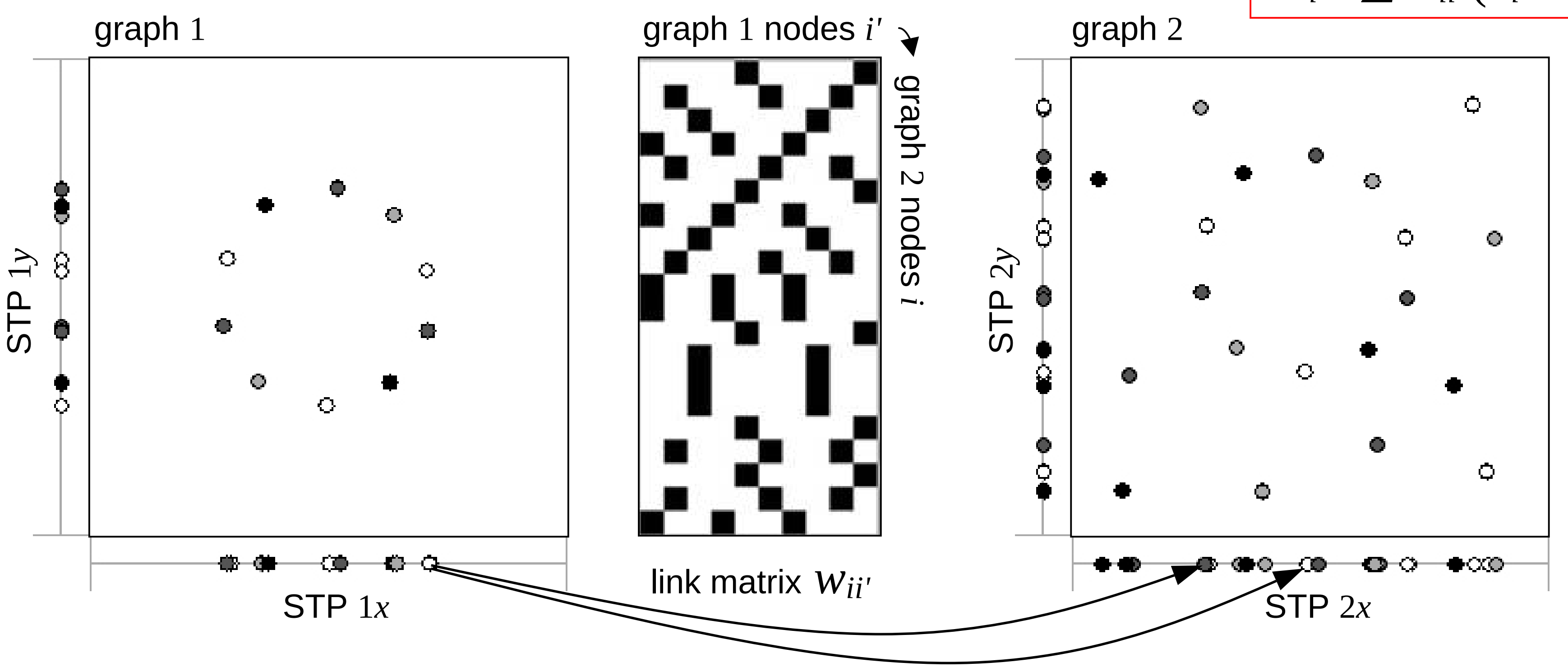
4. Dynamic Temporal Matching

➢ 1-D dynamic phase matching

✓ graph matching implemented as dynamical link matching between two pairs of STPs

$$\begin{cases} \dfrac{du_i}{dt} = c(u_i - \dfrac{u_i^3}{3} + v_i + z) + \eta + K_i + W_i \\ \dfrac{dv_i}{dt} = \dfrac{1}{c}(a - u_i - bv_i) + \eta \end{cases}$$

$$W_i = \sum w_{ii'}(u_{i'} - u_i)$$



4. Dynamic Temporal Matching

➢ 1-D dynamic phase matching

- ✓ additional coupling term: $W_i^{Xx}(t) = \sum_{\substack{j=1 \\ u_{i'}^x(t)<0}}^{N} w_{ii'}(t)\Big(u_{i'}^x(t) - u_i^X(t)\Big)$
- ✓ where $w_{ii'}$ varies according to

1. Hebbian-type synaptic plasticity based on temporal correlations

$$\Delta w_{ii'}(t) = \alpha\Big(- w_{ii'}(t) + w_0 f(s_{ii'}^{Xx}(0))\Big) \quad \text{with}$$

$$s_{ii'}^{Xx}(0) = \langle u_i^X(t')\ u_{i'}^x(t')\rangle_{t-T_s}^{t} \quad \text{and} \quad f(s) = \big(1 + e^{-\lambda(s-s_0)}\big)^{-1}$$

2. competition: renormalize efferent links

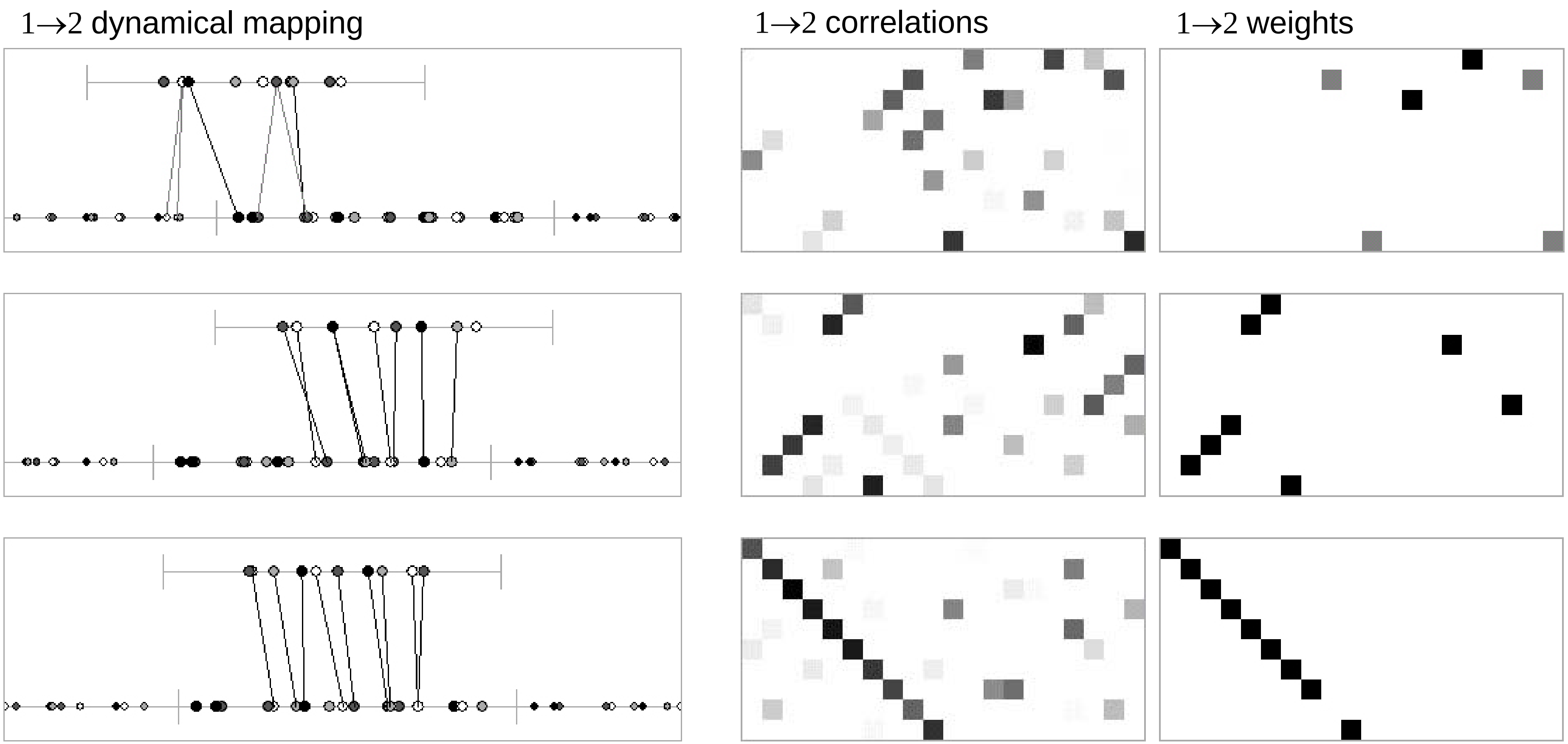
$$w_{ii'} \;\rightarrow\; w_{ii'} / \textstyle\sum_j w_{ji'}$$

3. label-matching constraint

4. Dynamic Temporal Matching

➢ 1-D dynamic phase matching

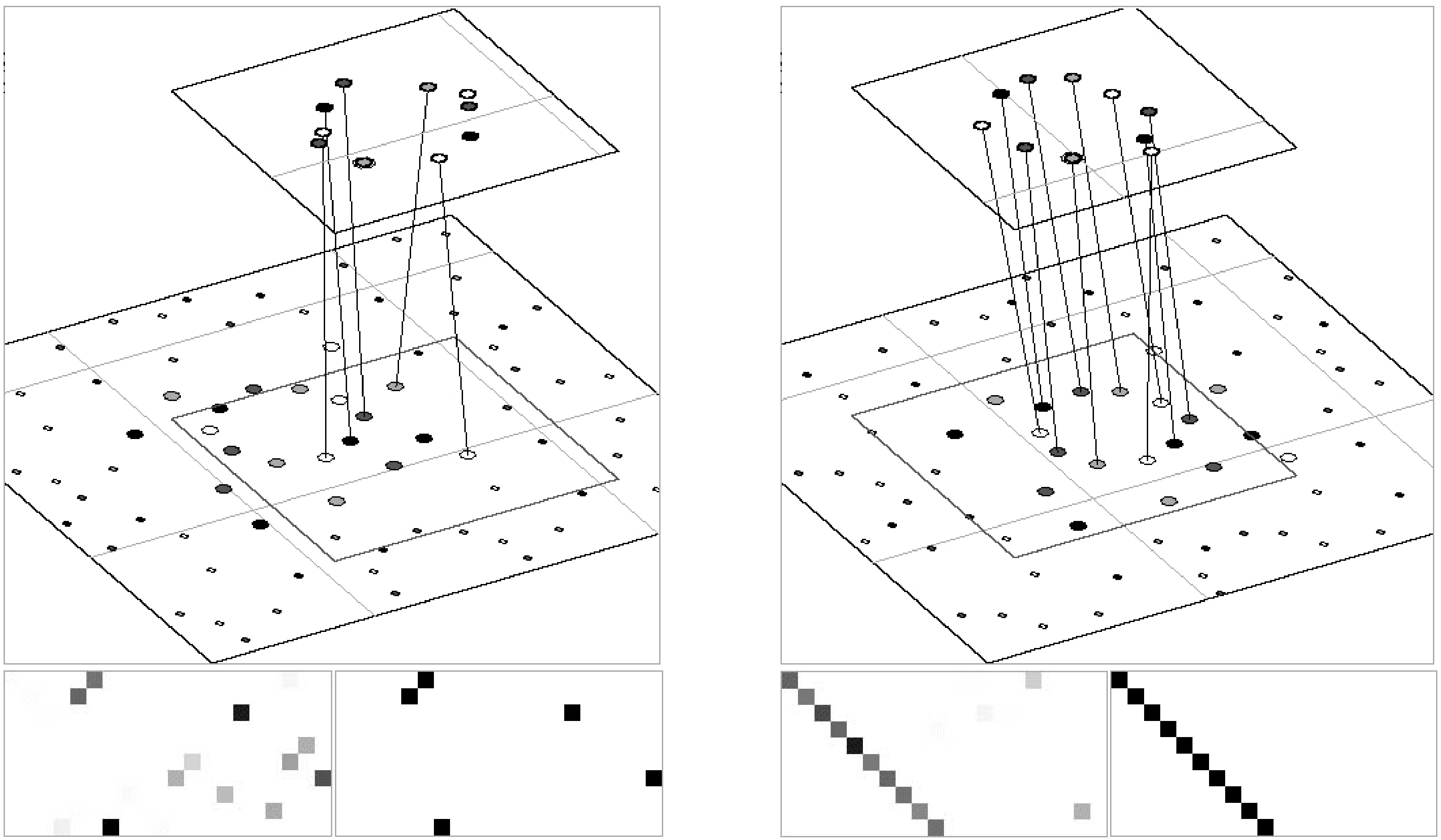
- ✓ labels and positions not constraining enough in 1-D: several possible partial matches (local minima)



4. Dynamic Temporal Matching

➢ 2-D dynamic phase matching

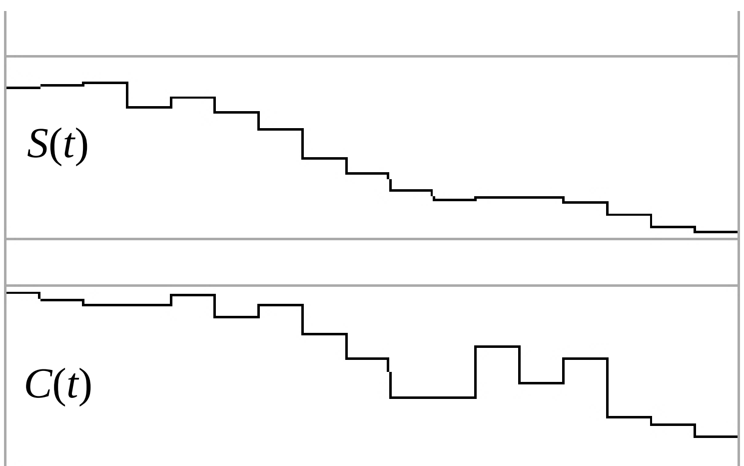
✓ Hebbian rule in 2-D: $\Delta w_{ii'}(t) = \alpha\Big(- w_{ii'}(t) + w_0 f\Big(\sqrt{s_{ii'}^{Xx}(0) s_{ii'}^{Yy}(0)}\Big)\Big)$



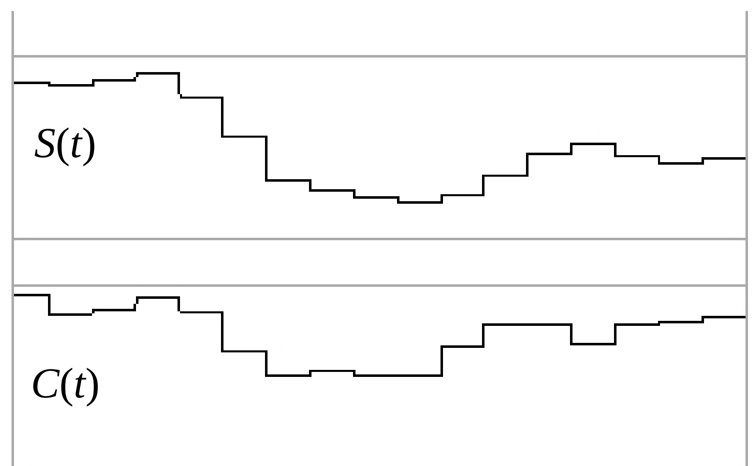
4. Dynamic Temporal Matching

➢ 2-D dynamic phase matching

- ✓ labels and positions more constraining in 2-D: less ambiguities
- ✓ however, to definitely find the best match (global minimum), we regularly drop and raise coupling strength within graph 2 layer
  - if match is weak, this will perturb STP 2 and undo matching links
  - if match is strong, this will not perturb STP 2 because it will be sustained by matching links → *resonance* between links and STPs



weak (mis)match → undone by uncoupling



strong match → resistant to uncoupling

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5. Lattice Wave Induction
   - Traveling waves
   - Wave induction
   - Dynamic wave mapping
   - Phase matching / elastic matching equivalence

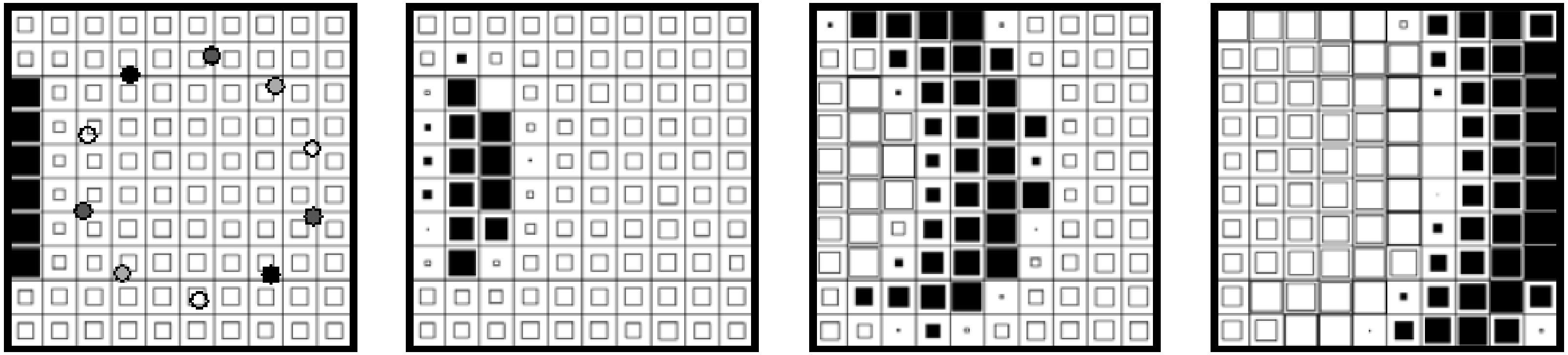
5. Lattice Wave Induction

➢ Traveling waves

- ✓ a constellation of dots can be a subset of a larger medium, analogous to a group of buoys on the water surface

→ *STP as a subset of a traveling wave on a lattice*

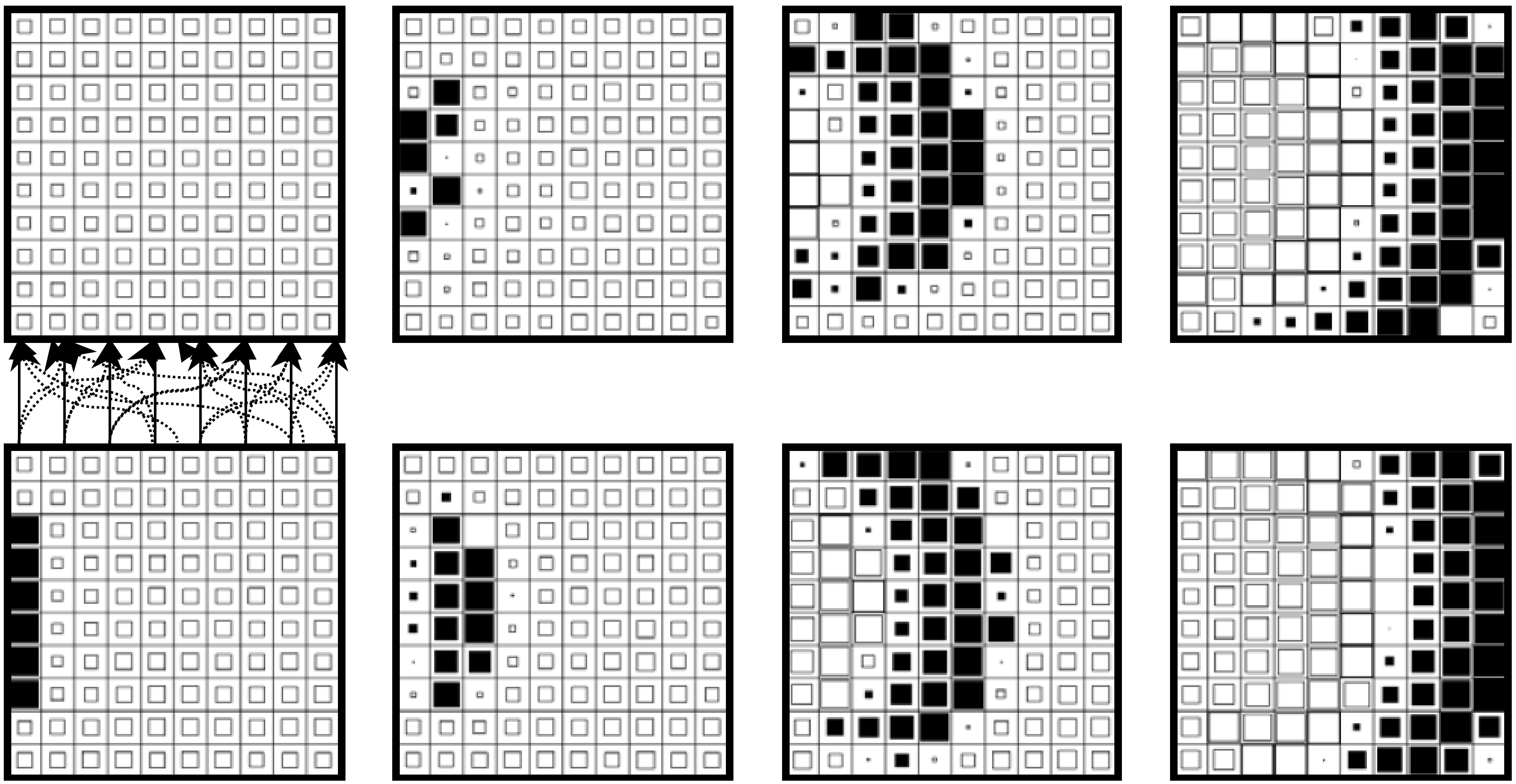
- ✓ instead of fully connected, transitive delays: *locally connected, uniform delays*



5. Lattice Wave Induction

➢ Wave induction

✓ through mapping links, a traveling wave on one layer can induce a wave on the other layer; all directions are possible



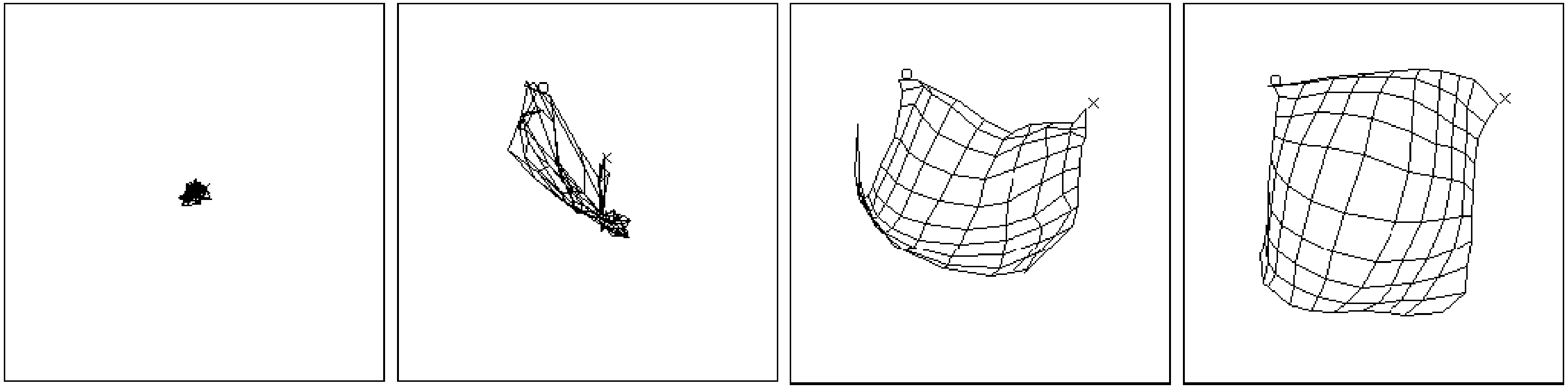
5. Lattice Wave Induction

➢ Dynamic wave mapping

✓ lattice wave induction is graphically equivalent to the unfolding of a 2-D mesh

→ *elastic matching fashion*

✓ each graphical point corresponds to a neighborhood average of destination phases, compounded by efferent mapping weights

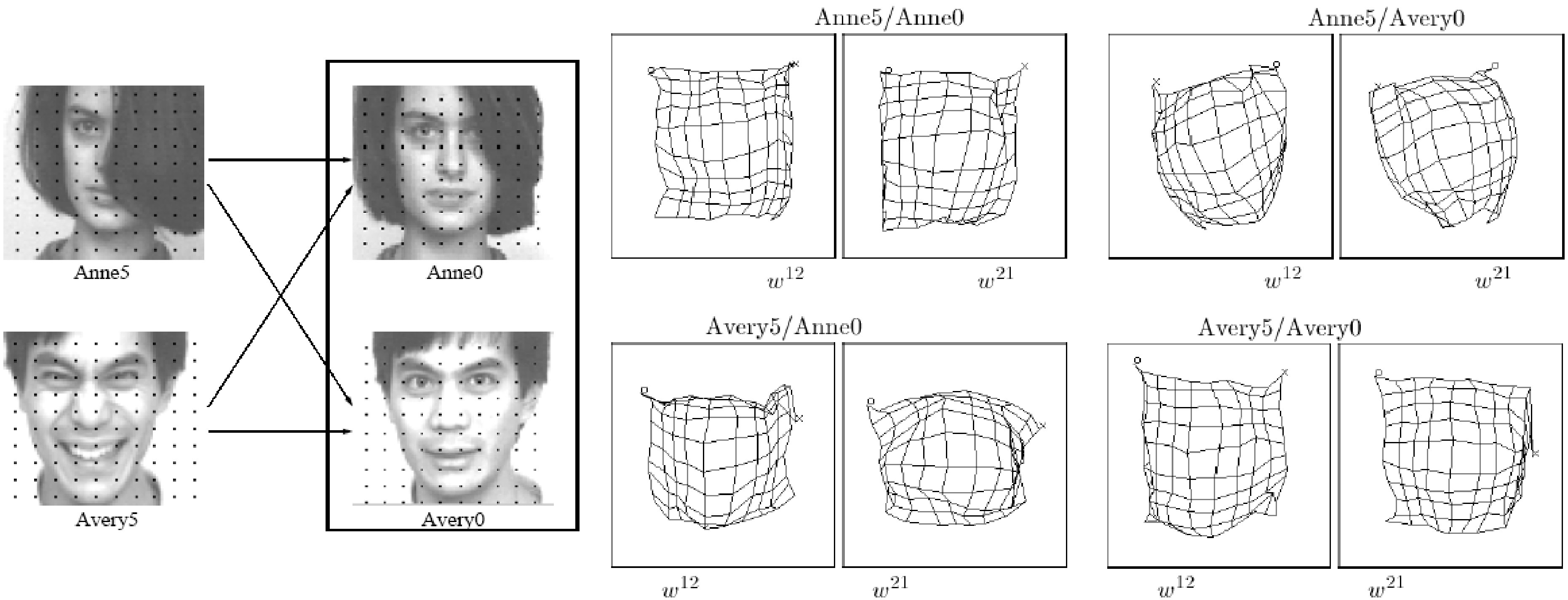


Schwarz, Andreas (1995), Technical Report (supervised by R. Doursat & L. Wiskott), Institut fuer Neuroinformatik, Bochum

5. Lattice Wave Induction

➢ Dynamic wave mapping

- ✓ ex: application to face recognition
- ✓ labels are Gabor-filter "jets"



Schwarz, Andreas (1995), Technical Report (supervised by R. Doursat & L. Wiskott), Institut fuer Neuroinformatik, Bochum

5. Lattice Wave Induction

➢ Phase matching / elastic matching equivalence

✓ similarity between Kuramoto's phase equation and elastic matching

- Kuramoto: phases attract each other, trying to minimize discrepancy with given delay (generally through sine function)

$$\frac{d\varphi_i}{dt} = \sum_j k_{ij}\Gamma(\varphi_j - \varphi_i - \omega\tau_{ij}) + g_i(t)$$

- elastic matching: link destinations attract each other, trying to minimize discrepancy with given rigid length

$$E = \tfrac{1}{2}k_0 \textstyle\sum_{i,j} \|\, \vec{r}_i - \vec{r}_j - (\vec{r}_i^{\,0} - \vec{r}_j^{\,0})\, \|^2$$

$$\Leftrightarrow \quad \frac{d\vec{r}_i}{dt} = \epsilon k_0 \sum_j (\vec{r}_j - \vec{r}_i - (\vec{r}_j^{\,0} - \vec{r}_i^{\,0}))$$

Pattern Recognition by Wave-Matching

1. Active Perception
2. Graph Matching
3. Phase Tagging
4. Dynamic Temporal Matching
5. Lattice Wave Induction