



Animal Patterns

Kyle McDermott

CS 790R – Spring '06 (2/6/2006)

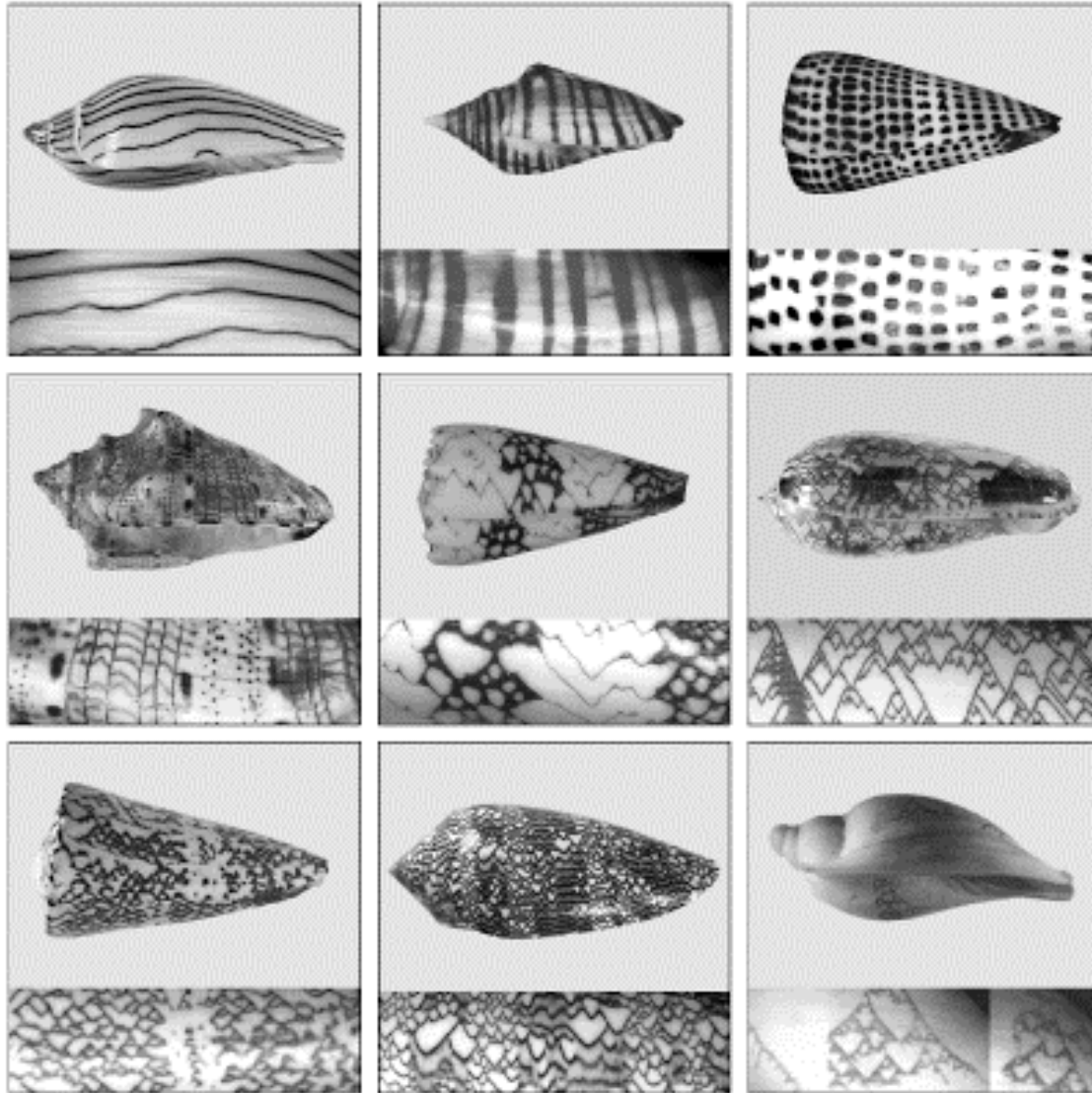
University of Nevada, Reno

Outline

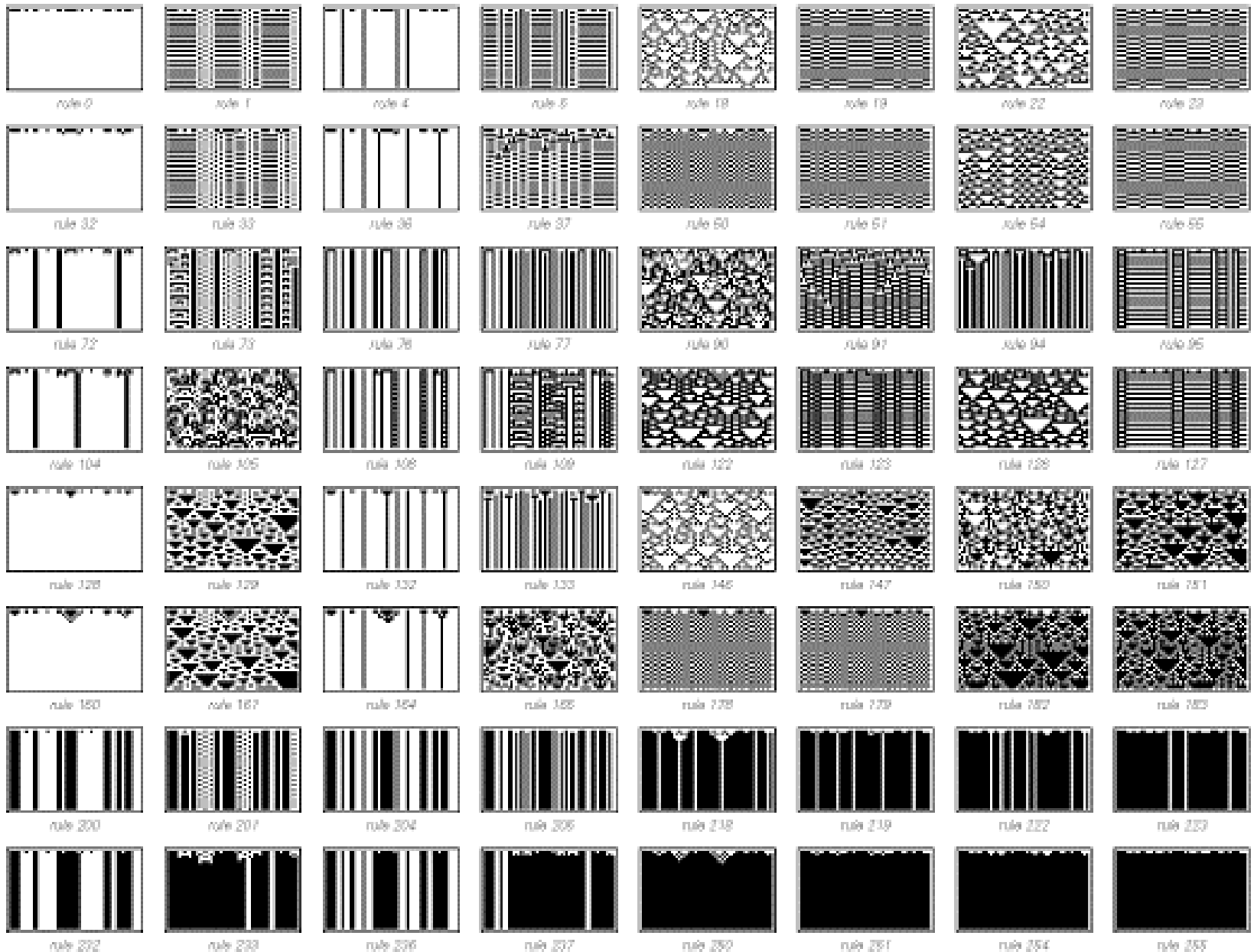
- Wolfram (2002) - *ever-so-briefly*
- Bar-Yam (1997) – Developmental Biology
 - Fur Demo
- Pearson (1993) – Complex patterns in a simple system
 - Texture Garden
- Mentioned:
 - Ball (1999) – Chapter: Bodies
 - Young (1984) – A local activator-inhibitor model of vertebrate skin patterns

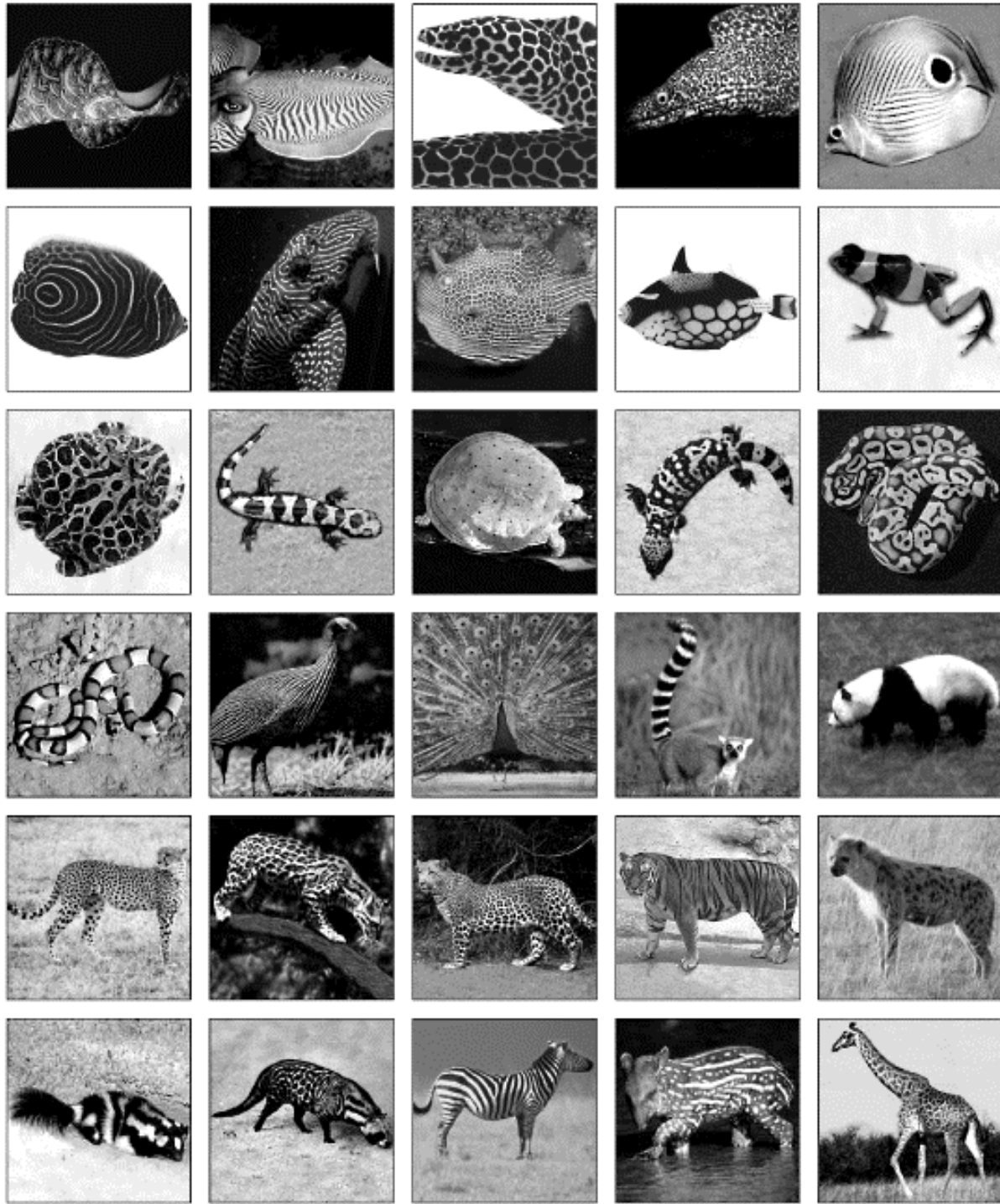
[The] New Kind of Science

(I describe in this book.)



1D Cellular Automata

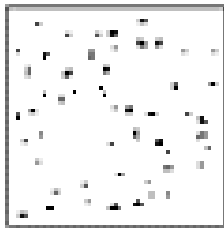




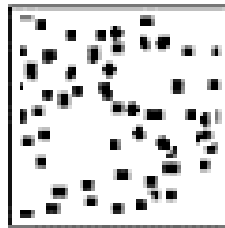
2D Cellular Automata



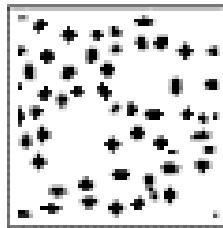
step 1



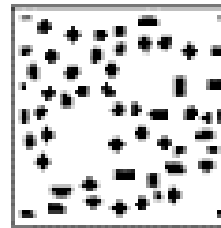
step 2



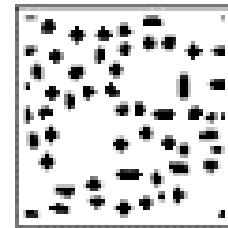
step 3



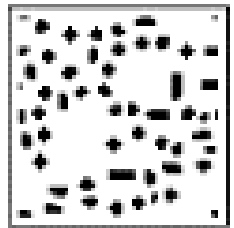
step 4



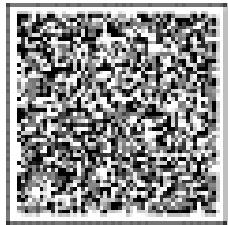
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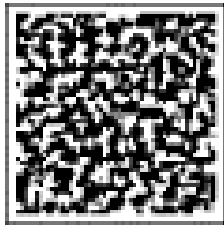
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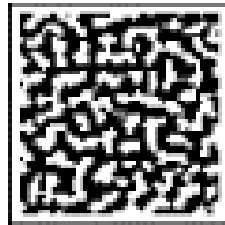
step 7



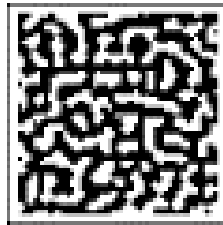
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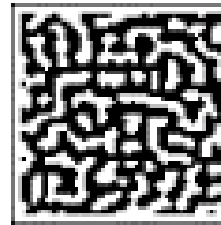
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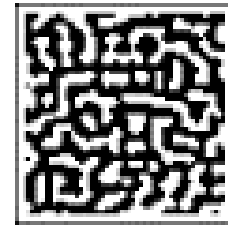
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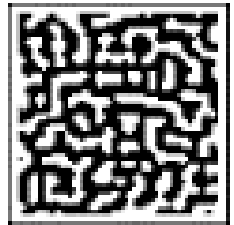
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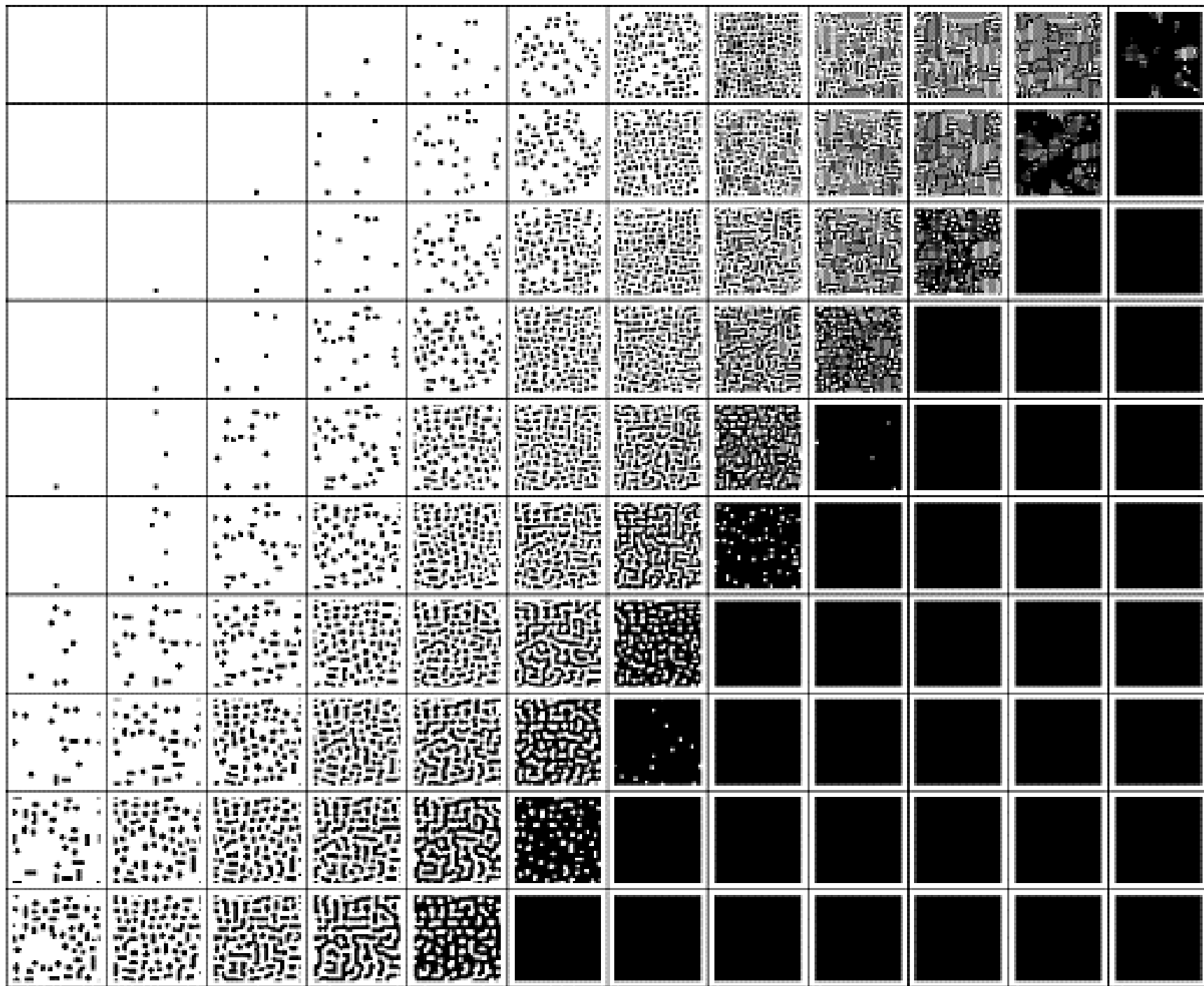
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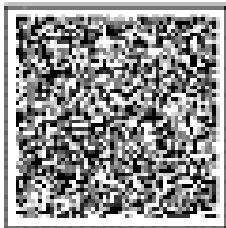
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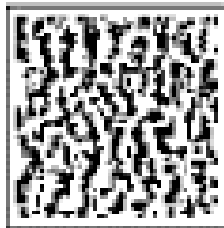
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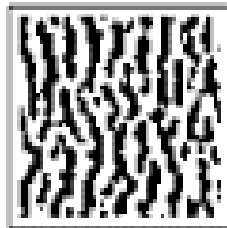
2D Cellular Automata - Stripes



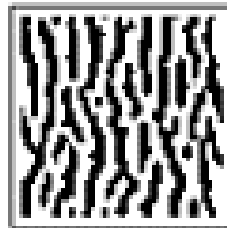
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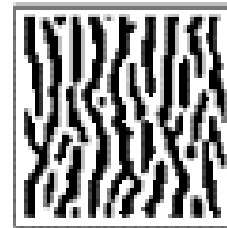
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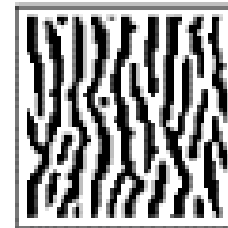
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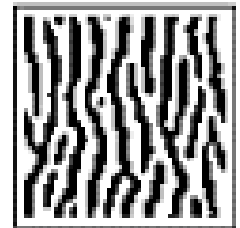
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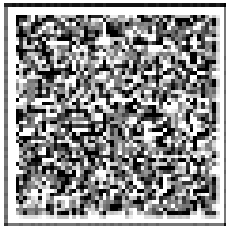
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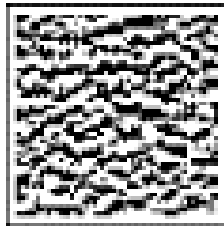
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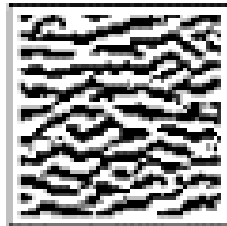
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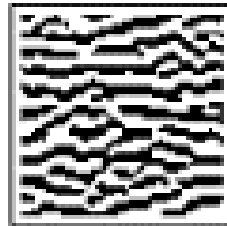
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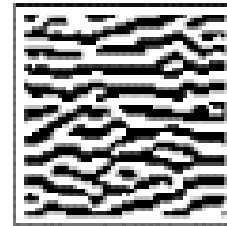
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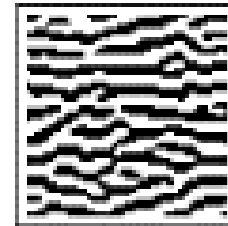
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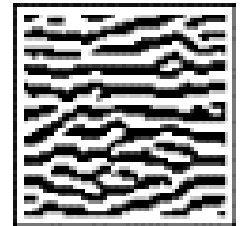
step 4



step 5



step 6



step 7

Bar-Yam (1997): Chapter 7

- 7.1 – Developmental Biology: Programming a Brick
- 7.2 – Differentiation: Patterns in Animal Colors
 - 7.2.1 – Introduction to pigment patterns
 - 7.2.2 – Activation and inhibition in pattern formation: CA models
 - 7.2.3 – Chemical diffusion
 - 7.2.4 – Chemical reactions
 - 7.2.5 – Pattern formation in reaction-diffusion systems



Developmental Biology

- How does an individual cell through cell division, differentiation, and growth result in an organism with complex physiology?
 - The program present within the cell
 - The environment in which it develops
- DNA contains many different instructions that are active or not depending on what the cell and its neighbors are doing

Nature of DNA function



- DNA is a collection of templates or blueprints for making protein chains
- The role of DNA at a particular time depends on which templates are being transcribed and which are not
- The activity of a template is determined by the activity of others

Cell behavior

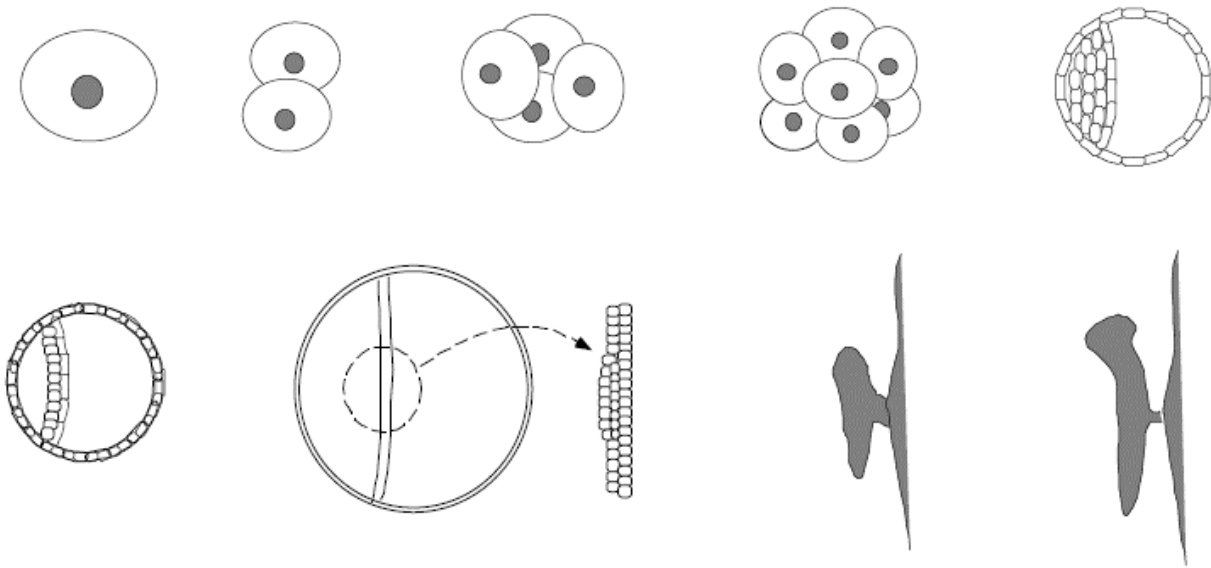
A decorative header element consisting of six circles arranged in a horizontal row. The first circle is solid light purple. The second circle is white with a light purple outline. The third circle is solid light purple. The fourth circle is white with a light purple outline. The fifth circle is solid light purple. The sixth circle is solid light purple.

- DNA in every cell contains all the information necessary to build and sustain the organism
- The molecules in the cell could be viewed as the society, acting upon each other and responding to the environment, that decide how to use this information



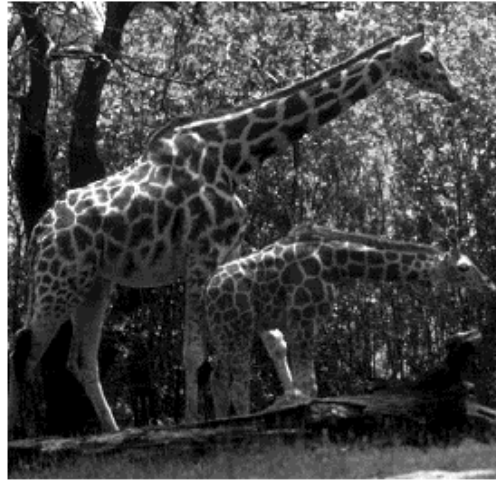
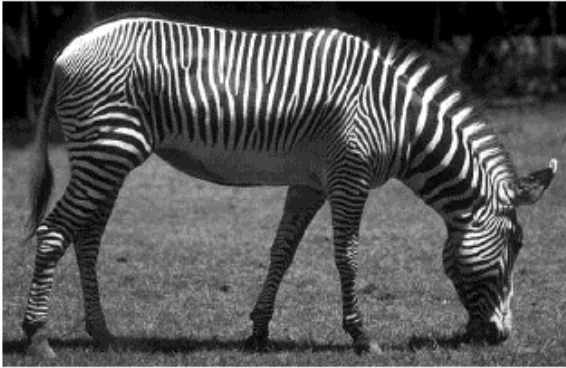
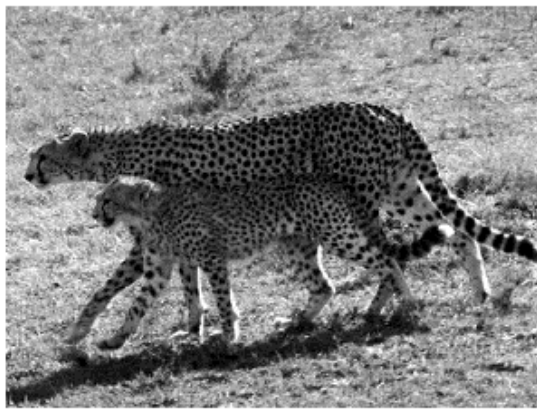
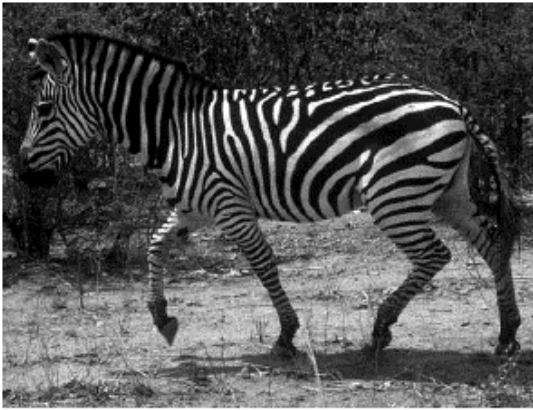
How would we construct a building?

- Create a program for a brick, describing how a brick should move and interact with other bricks around it
- Leave a pile of bricks all with the same program, and come back to find a finished building with windows, ducts, utilities etc. all in place

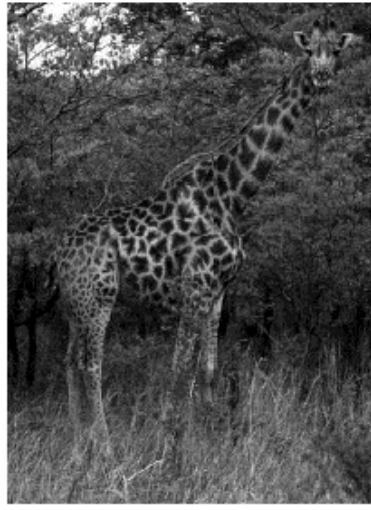
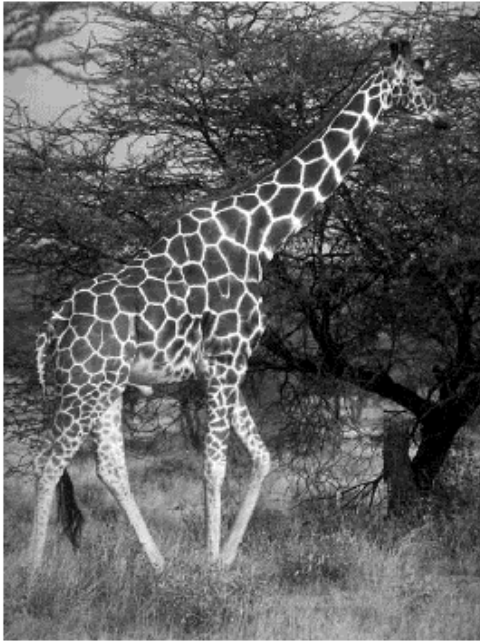


Patterns in Animal Colors

- Natural selection can be seen as a large influence in many cases
 - Uniform color to match a well-defined environment
 - Prey animals camouflaged to the environment
 - Prey animals camouflaged to each other
 - Predators can identify others of their species, perhaps identify individuals by forms within their own pattern
- Key feature – Differentiation



- Spots
- Stripes
- Polygons?





Activation and inhibition: CA models

- In a matrix of cells, each cell can communicate to its neighbors through emitted chemicals
- The range of communication depends on:
 - The diffusion constant of the chemical
 - Chemical reactions that may affect it
- A cell producing pigment that signals others to produce pigment is activating
- A cell signaling others to not produce pigment is inhibiting

Activation and inhibition: CA models

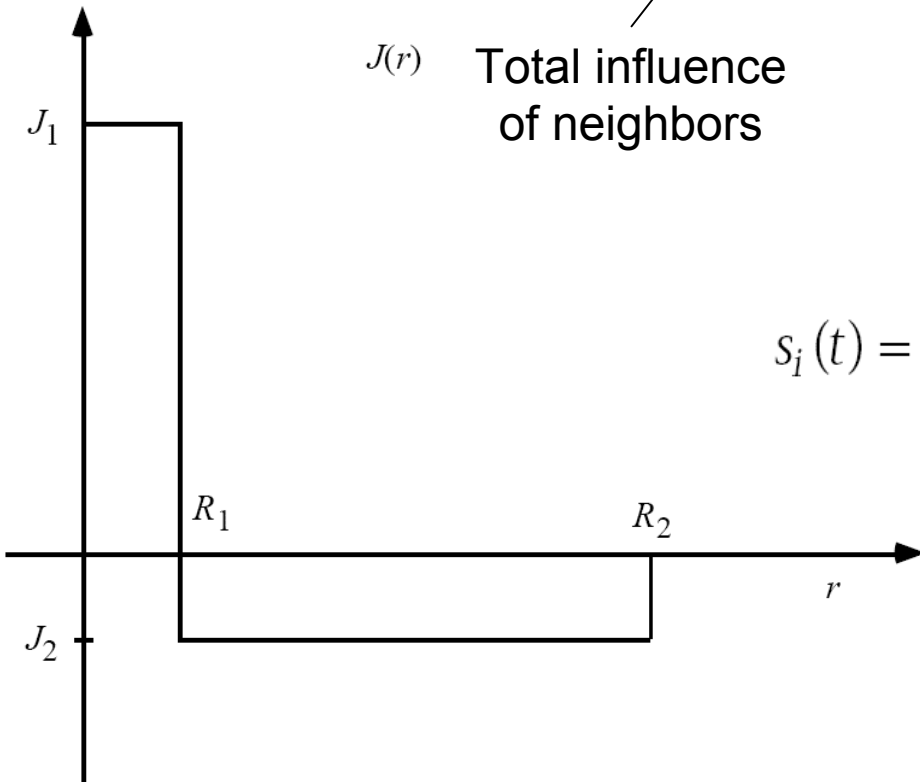
$$h_i = h + J_1 \sum_{r_{ij} < R_1} s_j + J_2 \sum_{R_1 < r_{ij} < R_2} s_j$$

$J(r)$ Total influence of neighbors

Uniform bias field

Activation from nearby cells

Inhibition from distant cells



$$s_i(t) = \text{sign} \left(h + J_1 \sum_{r_{ij} < R_1} s_j(t-1) + J_2 \sum_{R_1 < r_{ij} < R_2} s_j(t-1) \right)$$

$s = +1$, Cell is pigmented

$s = -1$, Cell is not pigmented

of iterations 0



3



1



4



2

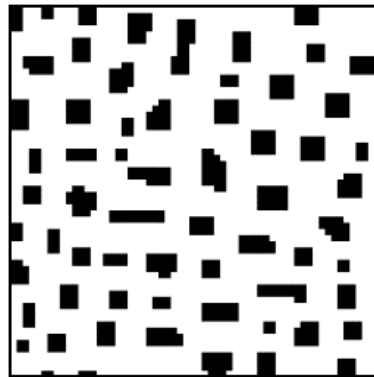
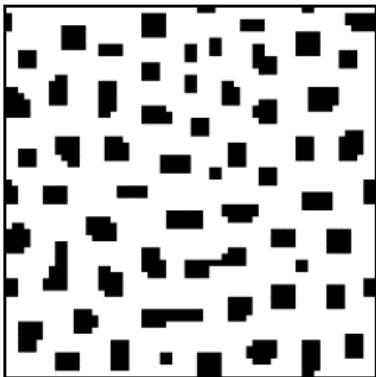


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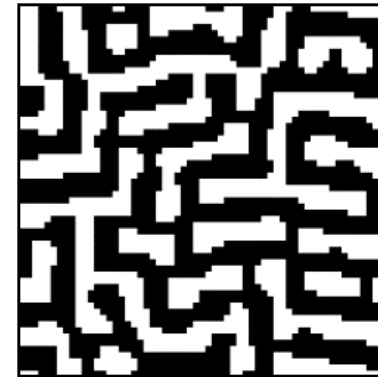


Uniform Bias Field h

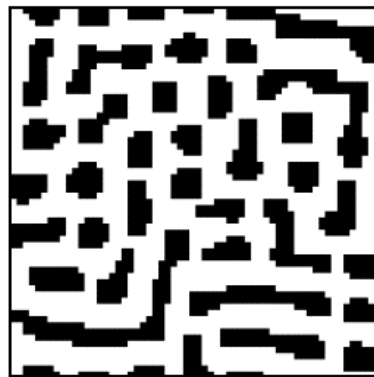
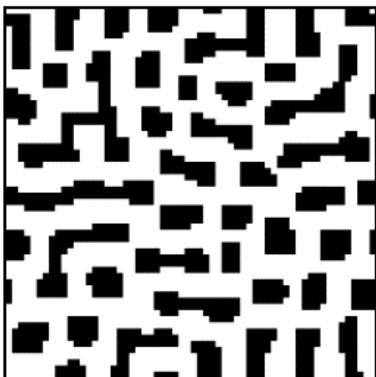
$h=-6$



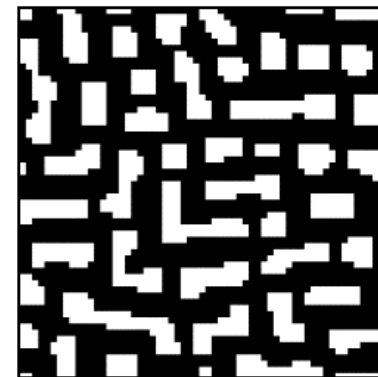
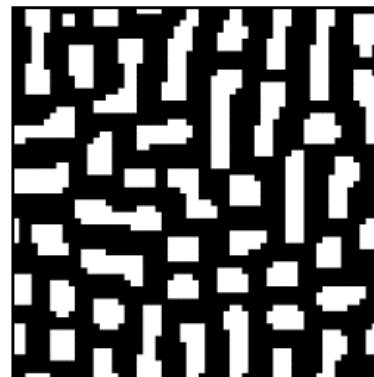
$h=1$



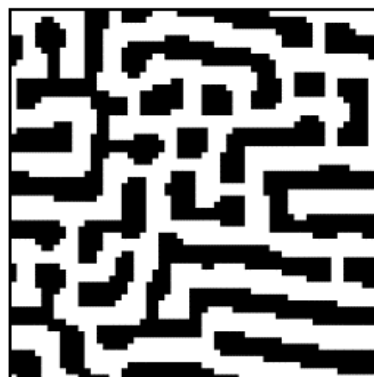
$h=-3$



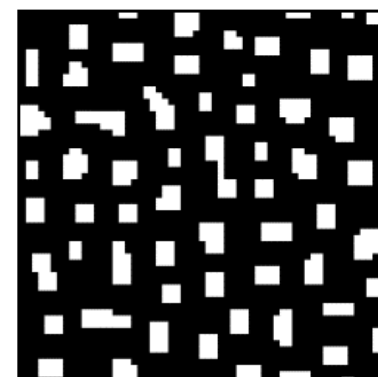
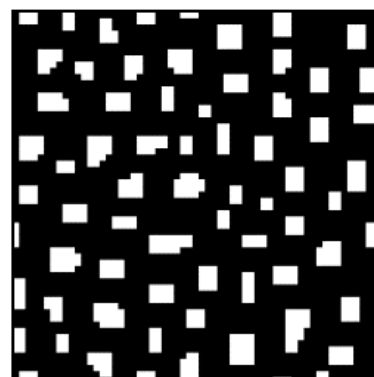
$h=3$



$h=-1$



$h=6$



Identical IC

Random IC

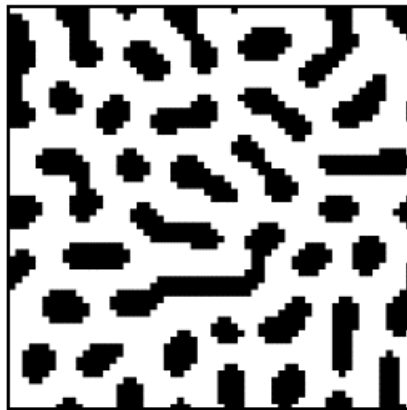
Identical IC

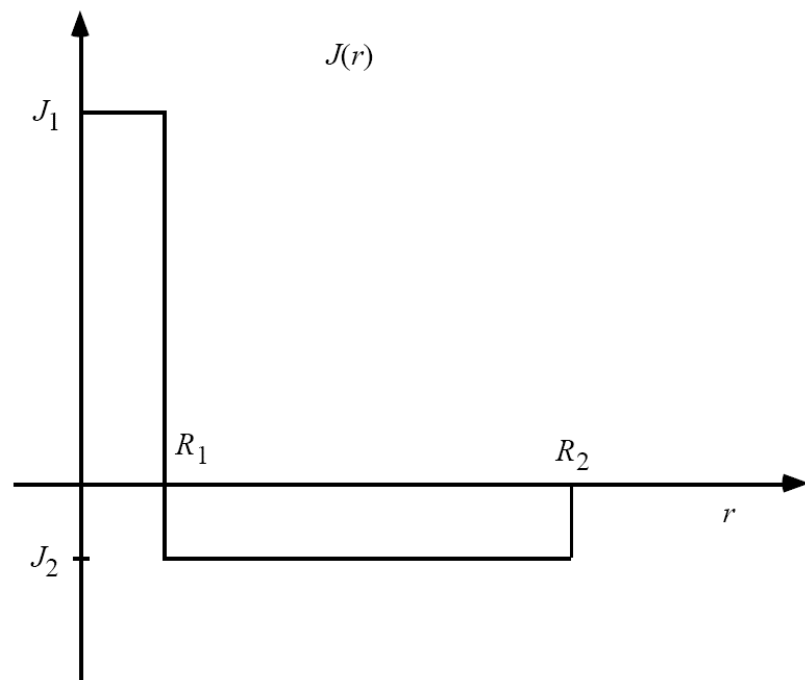
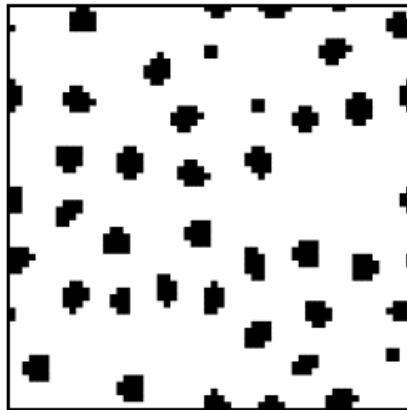
Random IC

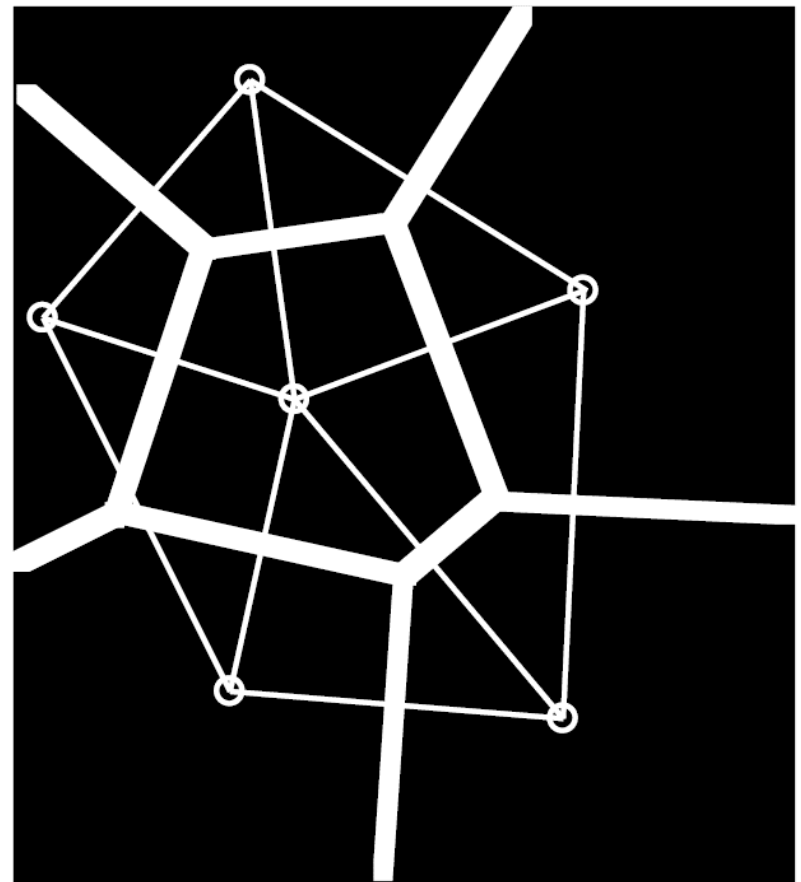
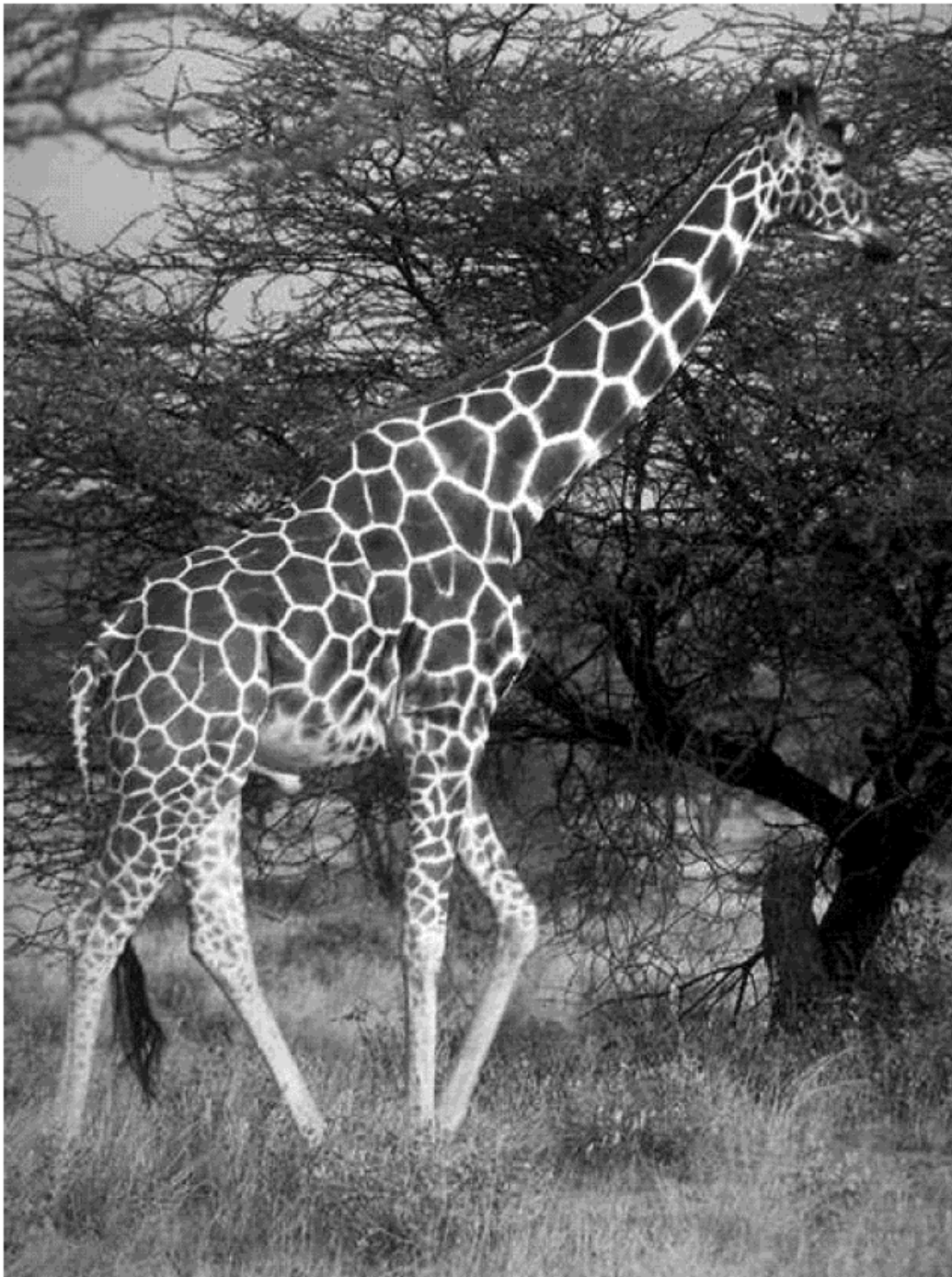
$R_2=6.0 \quad R_1=1.0 \quad h=0$

 $R_2=6.0 \quad R_1=1.5 \quad h=0$

 $R_2=7.0 \quad R_1=1.0 \quad h=0$

 $R_2=6.0 \quad R_1=1.5 \quad h=-3$

 $R_2=8.0 \quad R_1=1.0 \quad h=0$

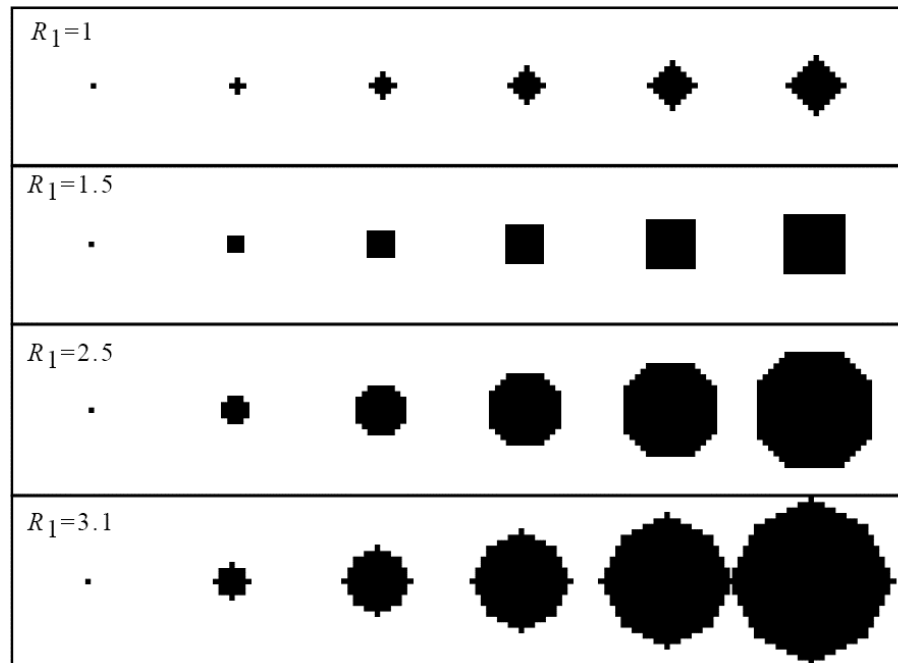
 $R_2=6.0 \quad R_1=1.5 \quad h=-6$




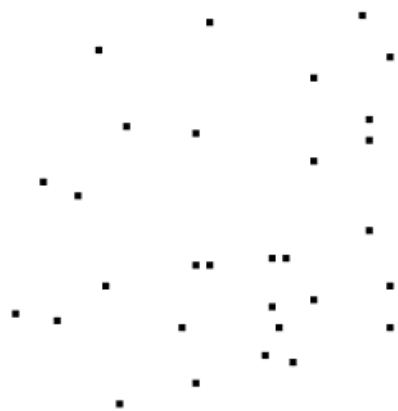
- First choose a sparse set of initial spots
- Divide the plane based on which spot each cell is closest to
- Add pigment within each

Generating the giraffe pattern

- Become pigmented at time t if at least one neighbor was pigmented at time $t - 1$
- Do not become pigmented if too many neighbors are already pigmented (cannot de-pigment)



of iterations 0



3



1



4



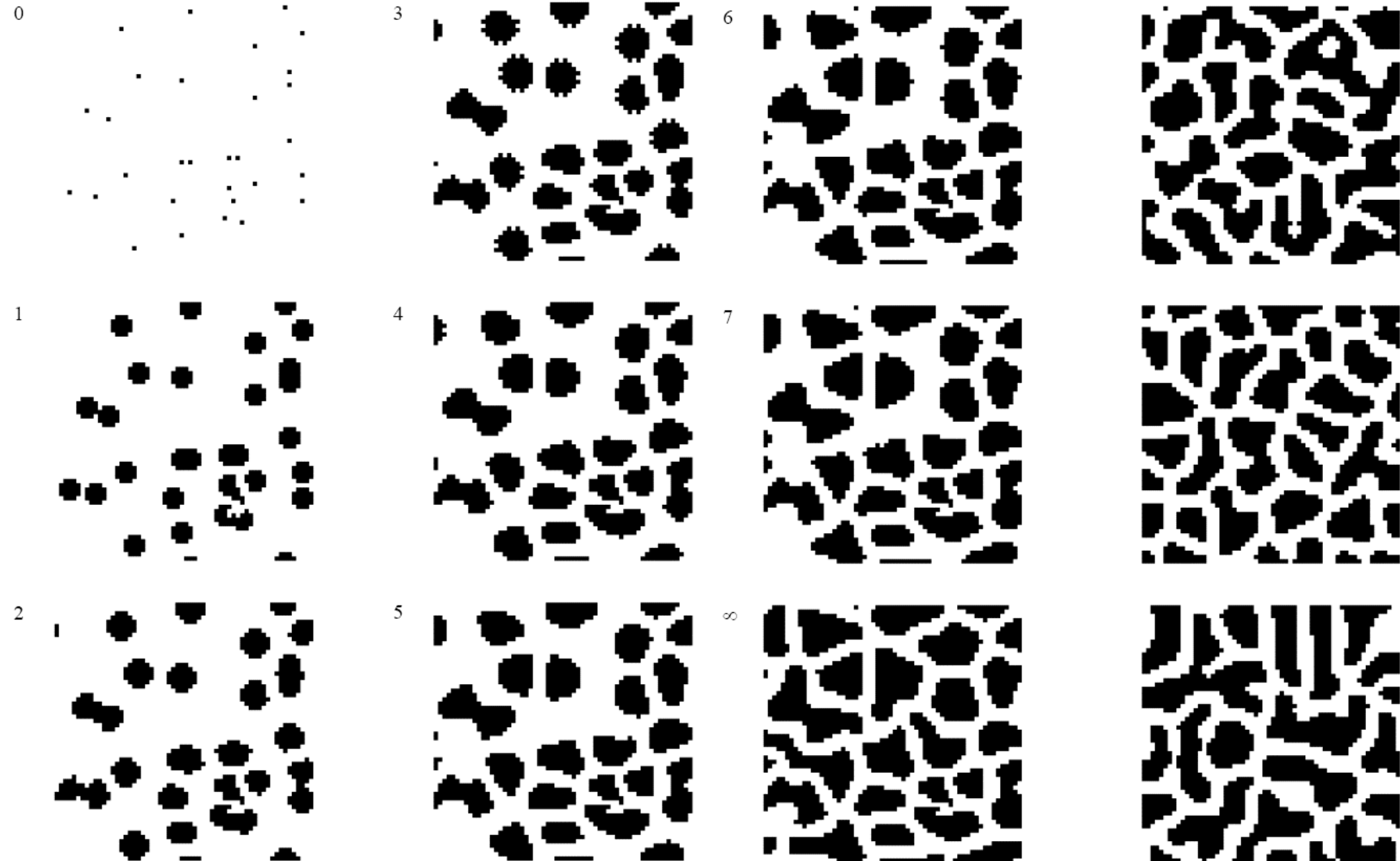
2



5

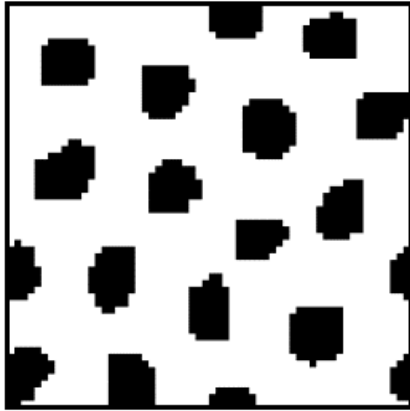


Introduce a second range, larger than that for growth, to
determine when to stop

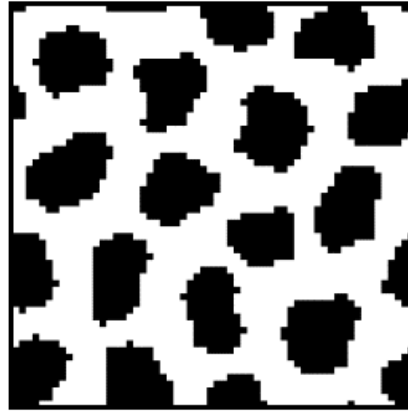


$R_2=12.0$ $R_1=1.0$ $h=-25$

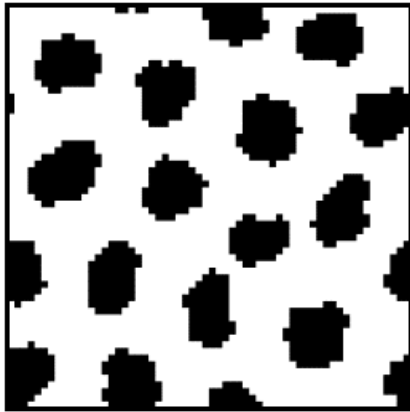
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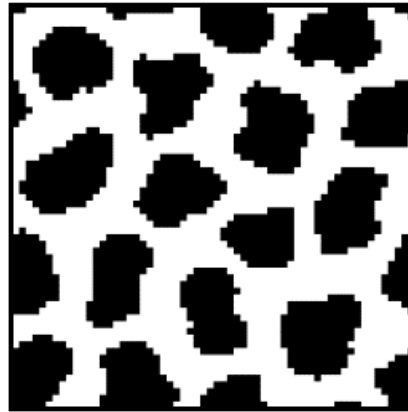
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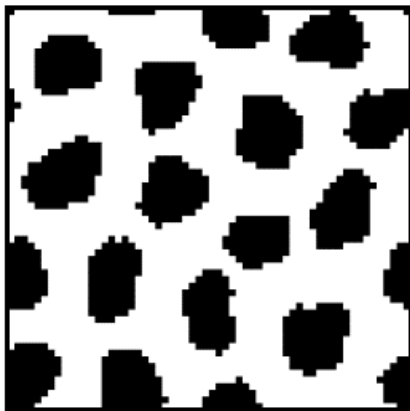
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4



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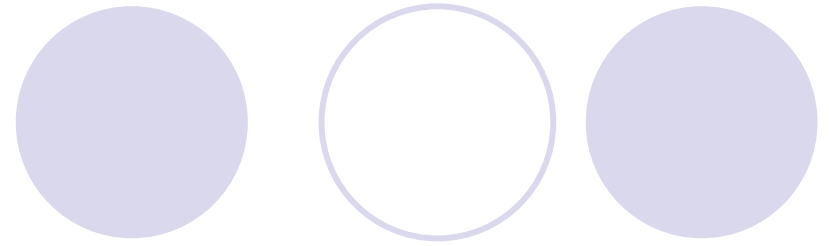


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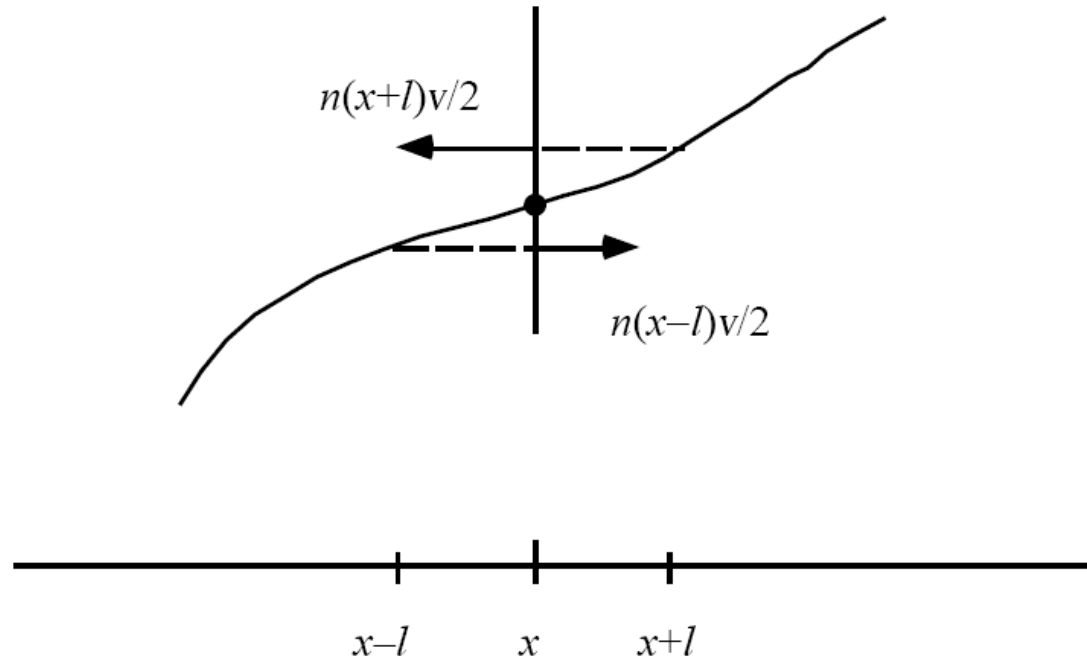
- Start with the earlier model using the parameters shown to generate evenly spaced spots
- Then grow out from those using the giraffe model

Chemical Diffusion



- Look at the motion of a low-density “gas” of molecules that have a varying density profile as a function of position
- Molecules undergo collisions with characteristics:
 - t – Time between collisions
 - v – The velocity
 - l – The travel distance ($= v^* t$)
 - Neither v nor t depend on the density of the particular type of molecule

Chemical Diffusion



$$J(x) = \frac{v}{2} (n(x-l) - n(x+l)) \approx -lv \frac{dn(x)}{dx} = -v^2 \tau \frac{dn(x)}{dx}$$

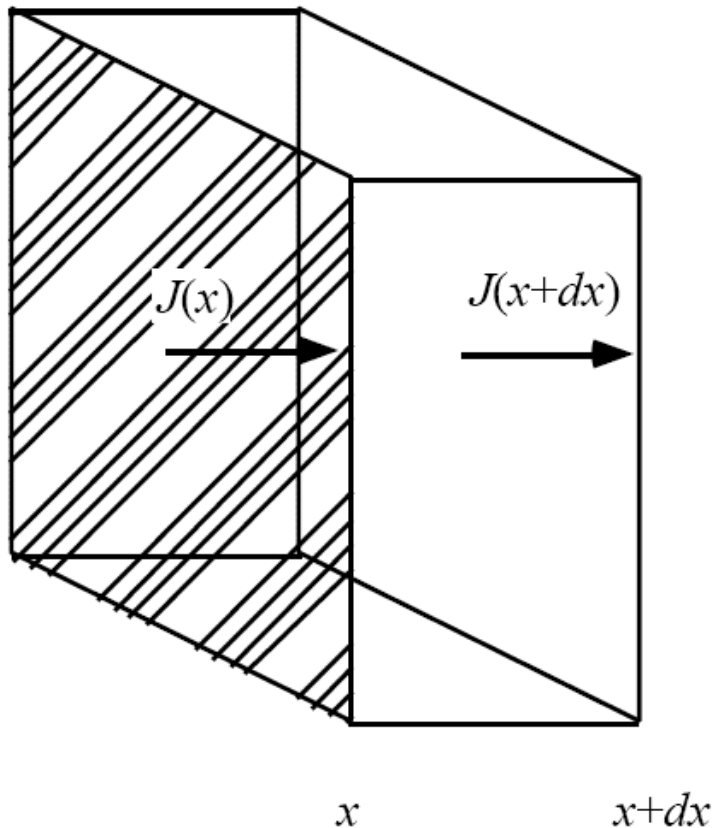
Current at
any point x

Density of particles coming
from either direction * velocity

This guy

Chemical Diffusion

$$\Gamma \Delta x \frac{dn(x;t)}{dt} = (-J(x + \Delta x/2;t) + J(x - \Delta x/2;t))\Gamma = -\Gamma \Delta x \frac{dJ(x;t)}{dx}$$



Gamma Delta x terms cancel,
substitute eq. for J:

$$\frac{dn(x;t)}{dt} = v^2 \tau \frac{d^2 n(x;t)}{dx^2}$$

Generalized to 3 dimensions:

$$\frac{dn(\mathbf{x};t)}{dt} = D \nabla^2 n(\mathbf{x};t)$$

Chemical Reactions

$$\frac{dn_i(x;t)}{dt} = D\nabla^2 n_i(x;t) + R_i(\{n_j(x;t)\})$$

- Introduce a new element in our eventual equation to account for chemical reactions in the system
 - Again use assumption of low densities
 - Fast reactions
 - Density not too rapidly varying in space (so that gradients need not be considered)
 - Interactions are short range

Chemical Reactions

- So the rate of a reaction: $A + B \rightarrow C$ is dependent upon the densities of the reactants, $n_A n_B$.
- The rate of change of the densities is:

$$\frac{dn_A}{dt} = -k_1 n_B n_A$$

$$\frac{dn_B}{dt} = -k_1 n_B n_A$$

Loss due to reaction based
on the concentrations of both

$$\frac{dn_C}{dt} = k_1 n_B n_A$$

Production based on same

Chemical Reactions

- The reverse reaction: $A + B \leftarrow C$
adds the parameters:

$$\frac{dn_A}{dt} = -k_1 n_B n_A + k_2 n_C$$

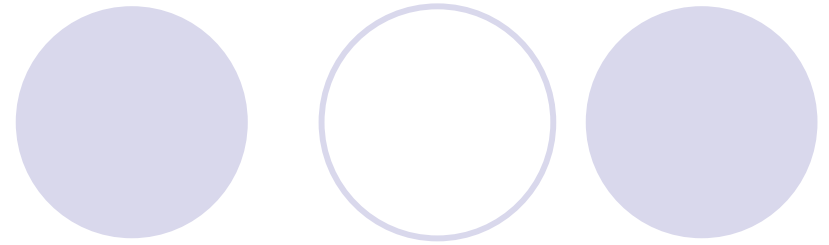
$$\frac{dn_B}{dt} = -k_1 n_B n_A + k_2 n_C$$

$$\frac{dn_C}{dt} = k_1 n_B n_A - k_2 n_C$$

Chemical Reactions

- The chemical reactions used are simplified and may each contain many steps not examined
- Certain assumptions can be made:
 - Quasi-equilibrium condition
 - Extreme kinetic regime
 - Quasi-static regime

Quasi-equilibrium



- If the two reversible reactions are in equilibrium ($A+B \rightarrow C$ and $C \rightarrow A+B$), then the density of A no longer changes with time, so: $n_B n_A = k'_2 n_C$ where $k'_2 = k_2/k_1$

Kinetic Regime

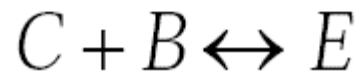
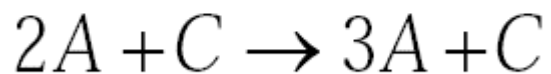
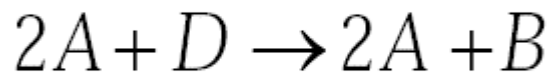
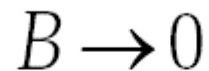
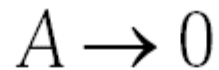
- If the densities are far from equilibrium, then the reaction can be seen as moving only in one direction

Quasi-static Regime

- If the density of one of the molecules is varying slowly on the time scale of observations
- The change to the density as compared to the density itself is negligible
- Example, if the density of C is large compared to all molecules in general then:

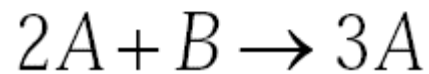
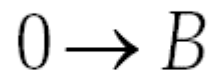
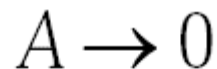
$$\frac{dn_A}{dt} \approx -k_1 n_B n_A + k_2'' \quad \text{where } k_2'' = k_2 n_C$$

Activator-inhibitor System



- 0 is used to denote a molecule whose density is irrelevant
- Most reactions assume extreme kinetic limit and only go in one direction

Activator-substrate System



- When 0 is used as a reactant, we assume the quasi-static case where it's concentration does not change significantly over the time of observation

Non-linear Dependence

- Take an example reaction: $2A \rightarrow B$

$$\frac{dn_A}{dt} = -k_3 n_A^2$$

$$\frac{dn_B}{dt} = k_4 n_A^2$$

- Where k_3 would be twice as large as k_4

Non-linear Dependence

- A more complex example that involves catalysis:

$$2A + D \rightarrow 2A + B$$
$$\frac{dn_D}{dt} = -k_5 n_A^2 n_D$$

$$\frac{dn_B}{dt} = k_5 n_A^2 n_D$$

- Where there is no change in the density of A in the reaction

Quasi-equilibrium Condition

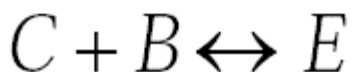
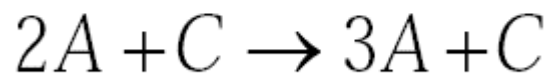
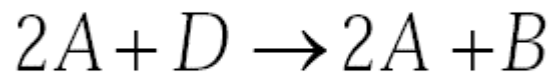
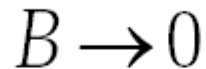
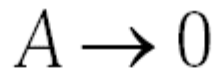
- Consider the reaction $C + B \leftrightarrow E$

$$n_B n_C = k'_2 n_E$$

- Then assume the density of E is large enough to remain relatively constant

$$n_C = \frac{k_2'''}{n_B}$$

Activator-inhibitor System



- Using the 1st and 4th equations (that affect the density of A) and the preceding assumption, we can construct:

$$\frac{dn_A}{dt} = -k_1 n_A + k_3 n_A^2 n_C \approx -k_1 n_A + k'_3 n_A^2 / n_B$$

Activator-inhibitor System

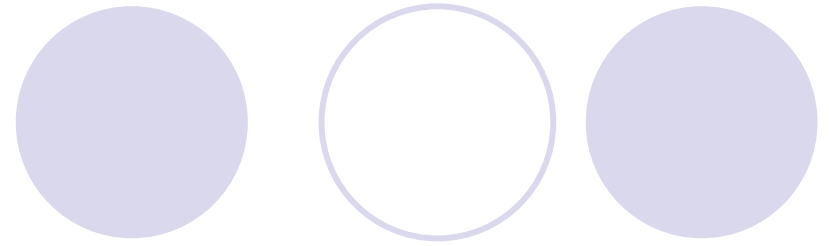
- For B we require another assumption, that $n_C \ll n_B$ to decouple the behavior of n_B from the quasi-equilibrium state

$$\frac{dn_B}{dt} = -k_4 n_B n_C + k_5 n_E + (k_3 n_A^2 n_D - k_2 n_B)$$

$$\frac{dn_C}{dt} = -k_4 n_B n_C + k_5 n_E$$

$$\frac{dn_B}{dt} = k_3 n_A^2 n_D - k_2 n_B$$

Pattern Formation



- Consider two types of molecules, an activator that acts over a short distance, and an inhibitor that acts over a longer distance
- Use A and B whose density $n_A(x,y,t)$ and $n_B(x,y,t)$ we write as $a(\mathbf{x};t)$ and $b(\mathbf{x};t)$
- These molecules diffuse with different diffusion constants D_a and D_b

Molecule Behavior

$$\frac{da(\mathbf{x};t)}{dt} = D_a \nabla^2 a(\mathbf{x};t) + f(a(\mathbf{x};t), b(\mathbf{x};t))$$

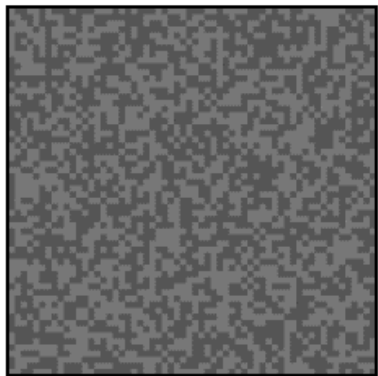
$$\frac{db(\mathbf{x};t)}{dt} = D_b \nabla^2 b(\mathbf{x};t) + g(a(\mathbf{x};t), b(\mathbf{x};t))$$

$$f(a, b) = k_1 a^2 / b - k_2 a$$

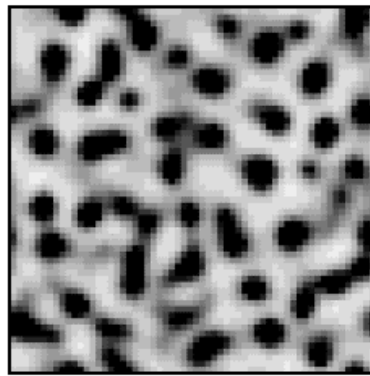
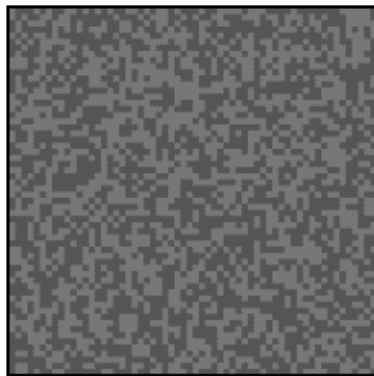
$$g(a, b) = k_3 a^2 - k_4 b$$

$$\frac{dn_A}{dt} = -k_1 n_A + k_3 n_A^2 n_C \approx -k_1 n_A + k'_3 n_A^2 / n_B$$

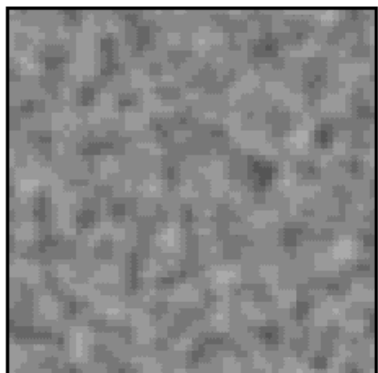
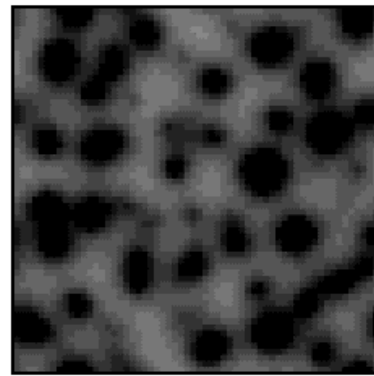
$$\frac{dn_B}{dt} = k_3 n_A^2 n_D - k_2 n_B$$



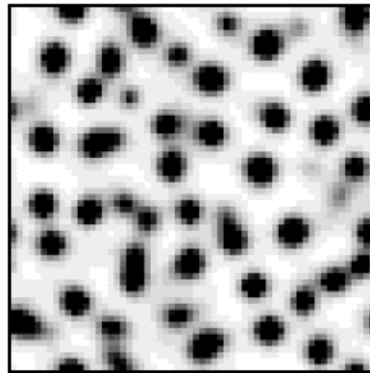
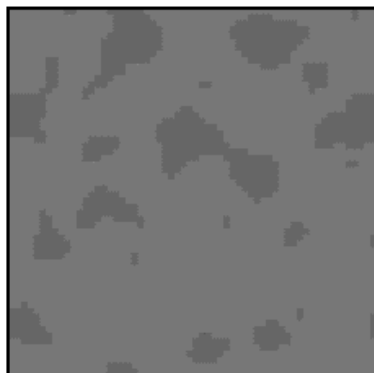
0



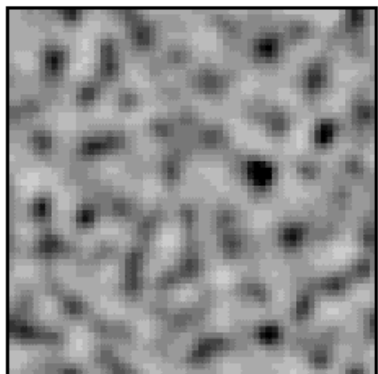
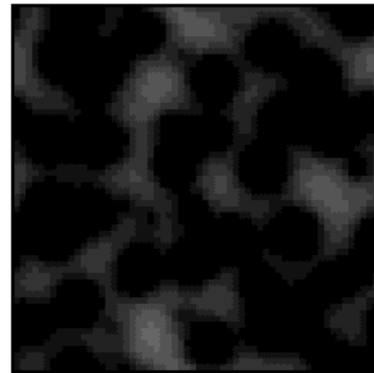
600



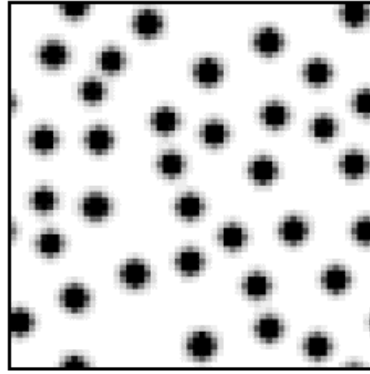
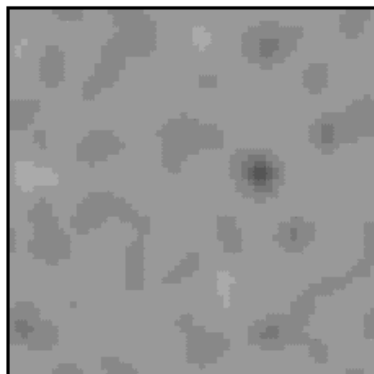
200



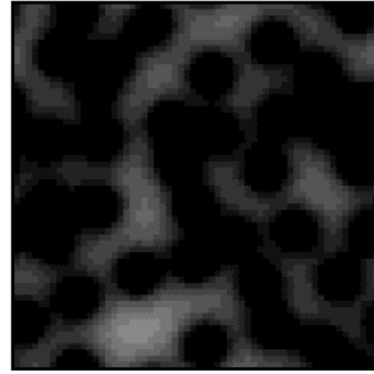
800



400



10,000



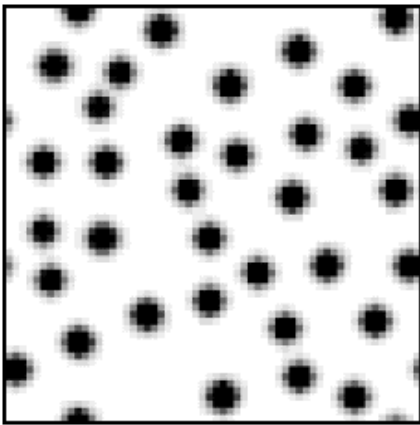
Activator A

Inhibitor B

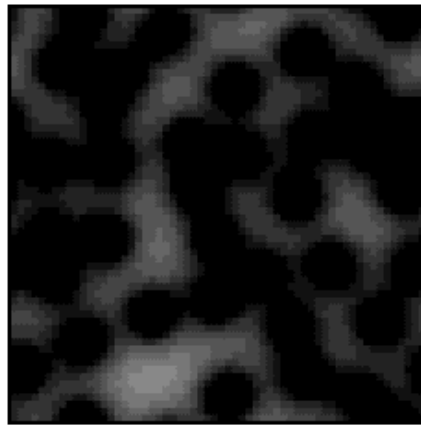
Activator A

Inhibitor B

$$k_1 = k_2 = k_3 = k_4 = 1$$



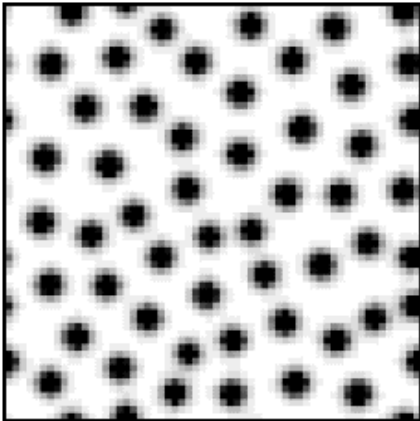
1:1



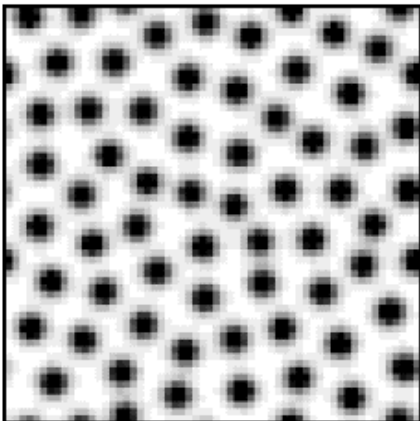
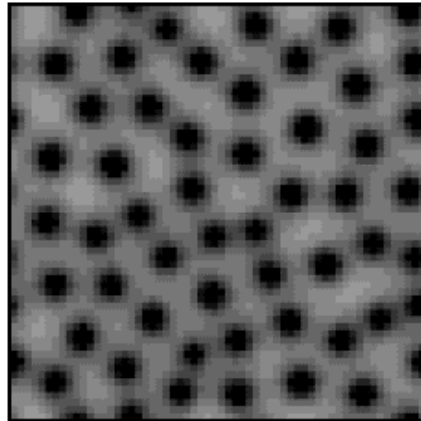
$$k_1 = k_2 = 1$$

$$k_3 = k_4 = 1, 2, 4$$

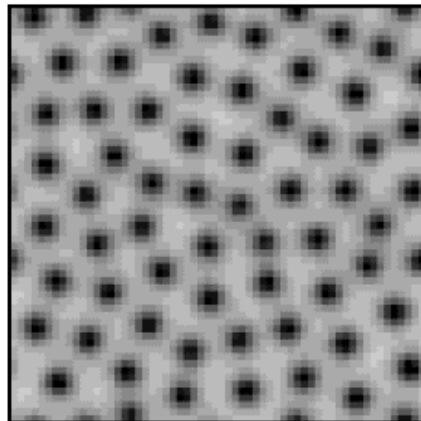
Since B is created by A they both have maxima at the same locations



1:2



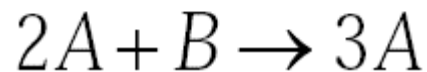
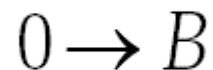
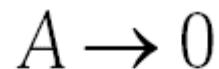
1:4



Activator-substrate system

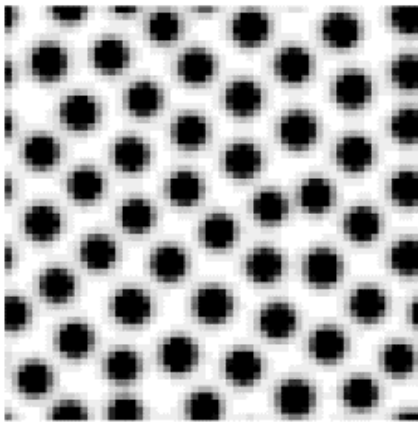
$$\frac{da(\mathbf{x};t)}{dt} = D_a \nabla^2 a(\mathbf{x};t) + f(a(\mathbf{x};t), b(\mathbf{x};t))$$

$$\frac{db(\mathbf{x};t)}{dt} = D_b \nabla^2 b(\mathbf{x};t) + g(a(\mathbf{x};t), b(\mathbf{x};t))$$

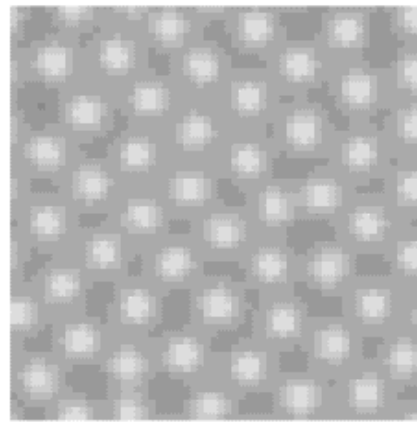


$$f(a,b) = k_1 a^2 b - k_2 a$$

$$g(a,b) = k_3 - k_4 a^2 b$$



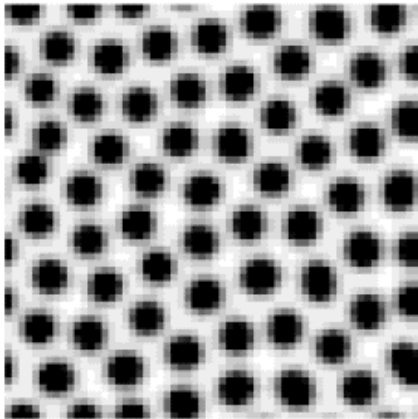
A-S
1:1



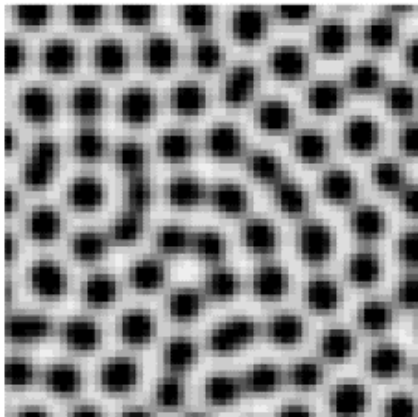
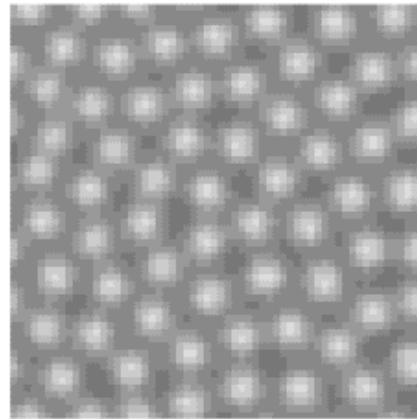
$$k_1 = k_2 = 1$$

$$k_3 = k_4 = 1, 2, 4$$

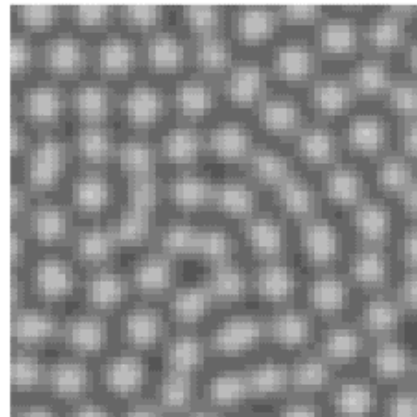
Since B is consumed by the creation of A the maxima of A correspond to the minima of B



A-S
1:2



A-S
1:4



Creating the Simulation



- Normalize the variables
 - Time and length scales relate to the time step and lattice size
 - Ultimately the number of independent parameters can be reduced to two (from 6 originally)
- Setting $f(a,b) = g(a,b) = 0$ provides a solution that is uniform but unstable, where any perturbation leads to the formation of patterns

Creating the Simulation

- Setting a and b initially to 1 sets $k_1 = k_2$ and $k_3 = k_4$, giving a range of values for the simulation between 0 (white) and 2 (black)
- Use a finite difference representation of the diffusion operator

$$\frac{d^2 a(x)}{dx^2} \rightarrow \frac{1}{\Delta x^2} (a(i+1) + a(i-1) - 2a(i))$$

Creating the Simulation

- In two dimensions this is:

$$\frac{d^2 a(\mathbf{x})}{dx^2} + \frac{d^2 a(\mathbf{x})}{dy^2} \rightarrow \frac{1}{\Delta x^2} (a(i+1, j) + a(i-1, j) + a(i, j+1) + a(i, j-1) - 4a(i, j))$$

- The time derivative represented as a time difference:

$$\frac{da(t)}{dt} \rightarrow \frac{1}{\Delta t} (a(t) - a(t-1))$$

Creating the Simulation

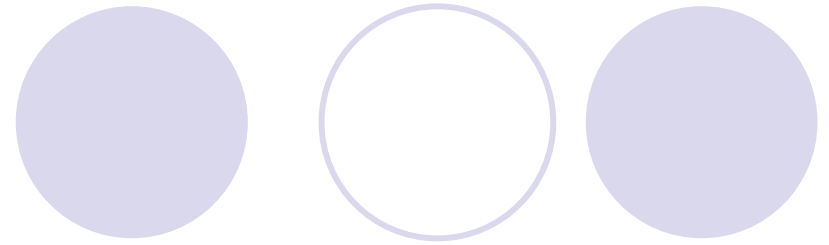
- Create a CA consistent with a random-walk model for molecular motion

$$a(i, j; t + 1) = a(i, j; t) + \Delta t f(a(i, j; t), b(i, j; t)) \\ + \frac{\Delta t}{\Delta x^2} D_a (a(i + 1, j; t) + a(i - 1, j; t) + a(i, j + 1; t) + a(i, j - 1; t) - 4a(i, j; t))$$

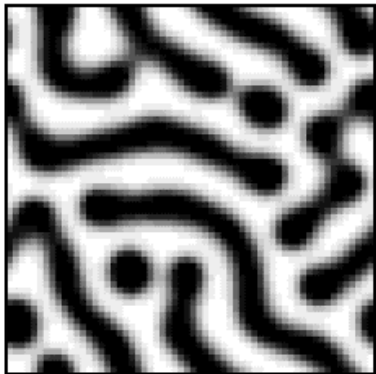
$$b(i, j; t + 1) = b(i, j; t) + \Delta t g(a(i, j; t), b(i, j; t)) \\ + \frac{\Delta t}{\Delta x^2} D_b (b(i + 1, j; t) + b(i - 1, j; t) + b(i, j + 1; t) + b(i, j - 1; t) - 4b(i, j; t))$$

- $\Delta t = 0.01$, $\Delta x = 1$, $D_a = 0.5$, $D_b = 20$

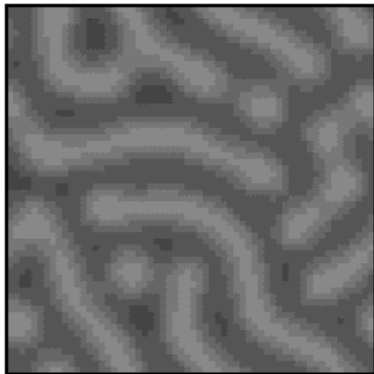
Making Stripes



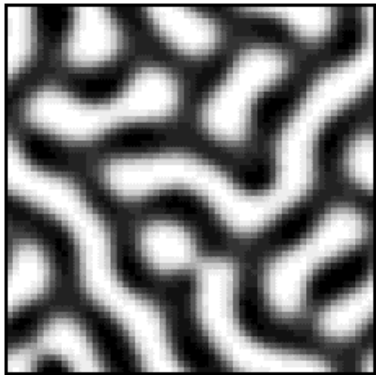
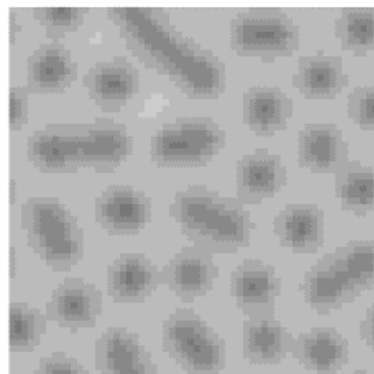
- Introduce a term that reduces activation at high values of a and maintains the inhibition at low values of a
- Activator-inhibitor $f(a,b) = k_1 a^2 / b(1 + k_5 a^2) - k_2 a$
 $g(a,b) = k_3 a^2 - k_4 b$
- Activator-substrate $f(a,b) = k_1 a^2 b / (1 + k_5 a^2) - k_2 a$
 $g(a,b) = k_3 - k_4 a^2 b / (1 + k_5 a^2)$



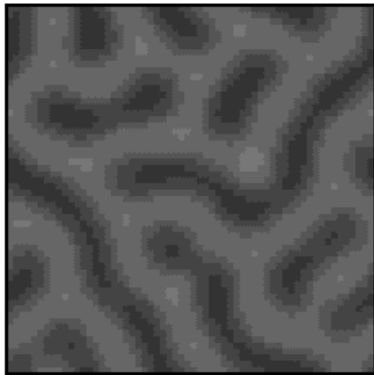
1:1
0.2



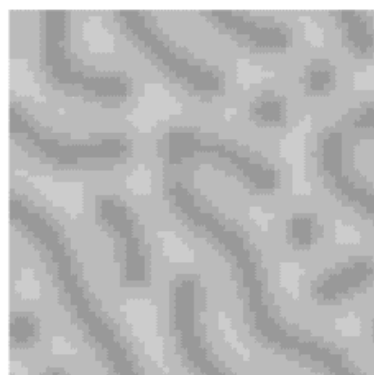
1:1
0.2



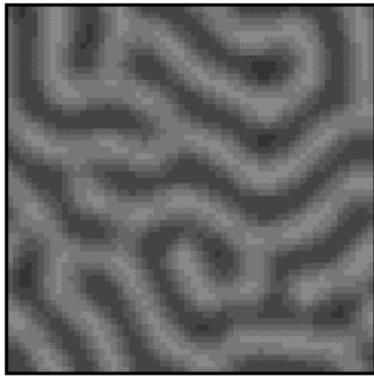
1:1
0.3



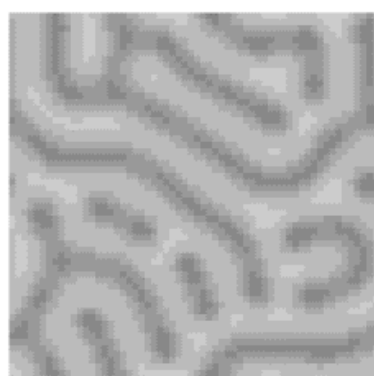
1:1
0.3



1:2
0.2



1:2
0.2

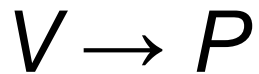


Pearson (1993)



- Look around in parameter space for a given reaction-diffusion model
- Observe some interesting behaviors
- Unfortunately in print one can only describe them
- “Texture Garden” demo

Gray-Scott Reaction-Diffusion Model



- Both reactions are irreversible

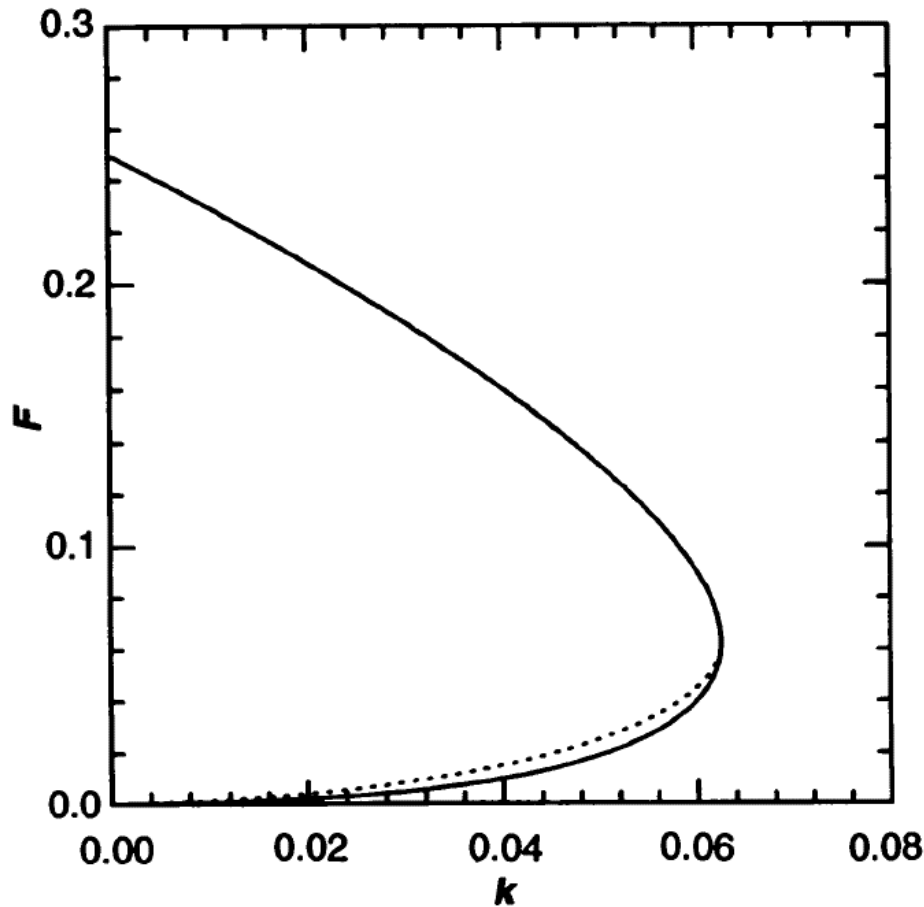
- P is an inert product

- Both U and V decay over time

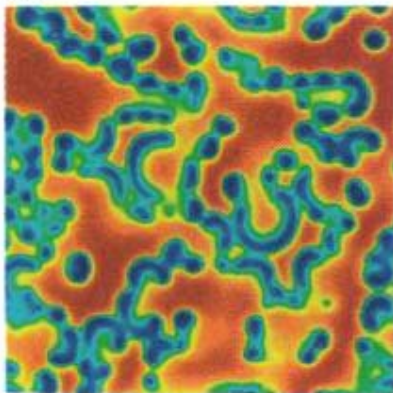
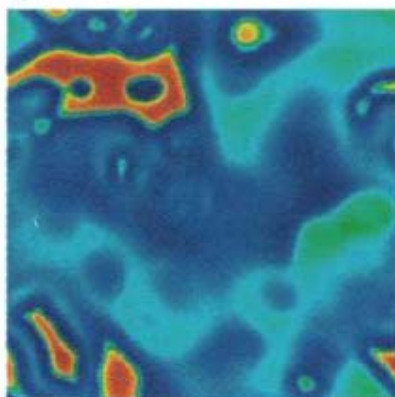
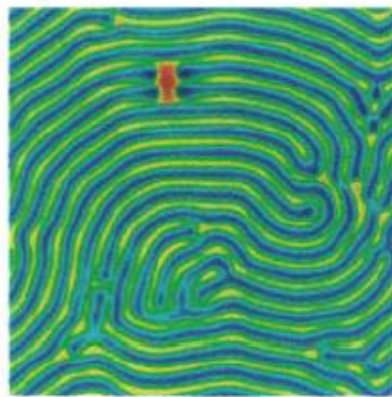
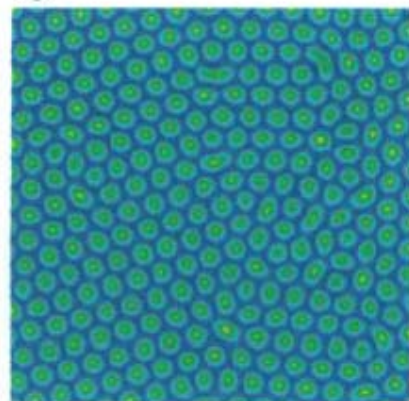
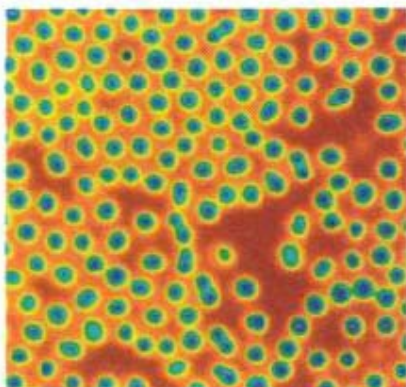
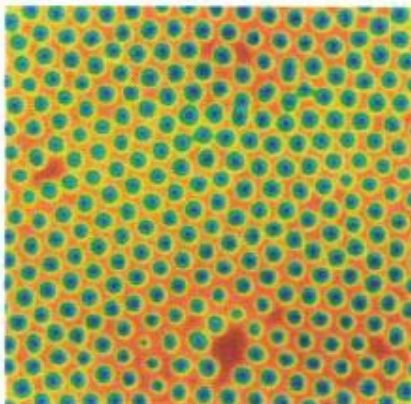
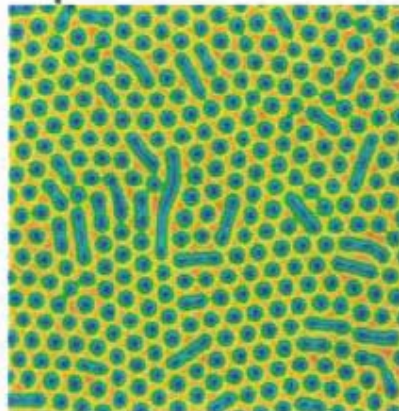
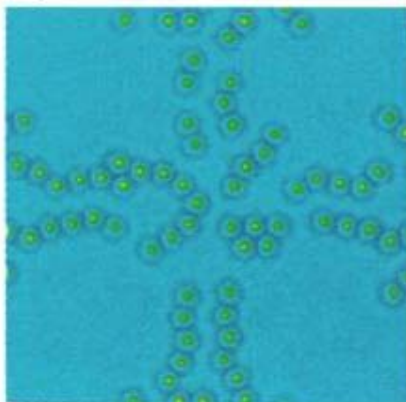
$$\frac{\partial U}{\partial t} = D_u \nabla^2 U - UV^2 + F(1 - U)$$

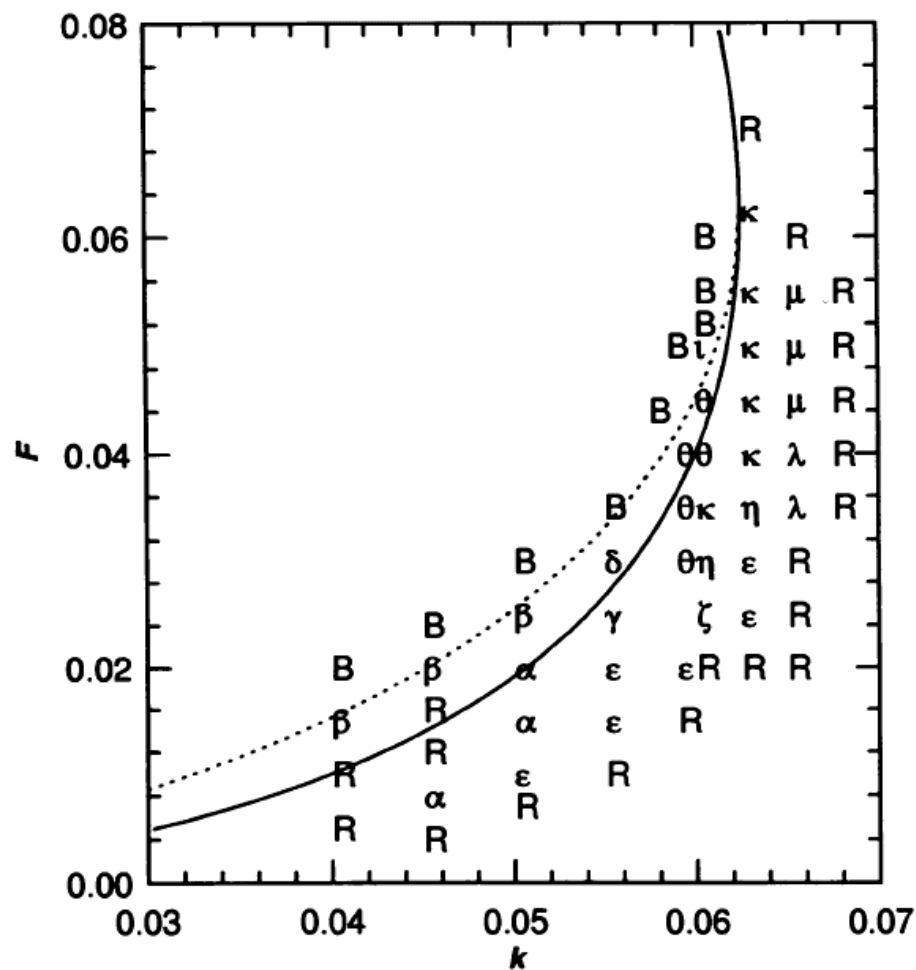
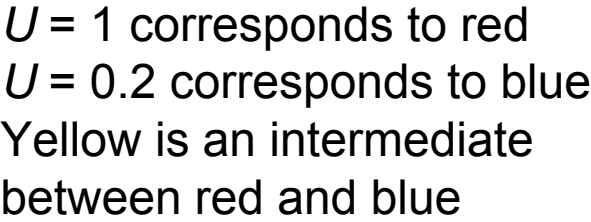
$$\frac{\partial V}{\partial t} = D_v \nabla^2 V + UV^2 - (F + k)V$$

Phase Diagram

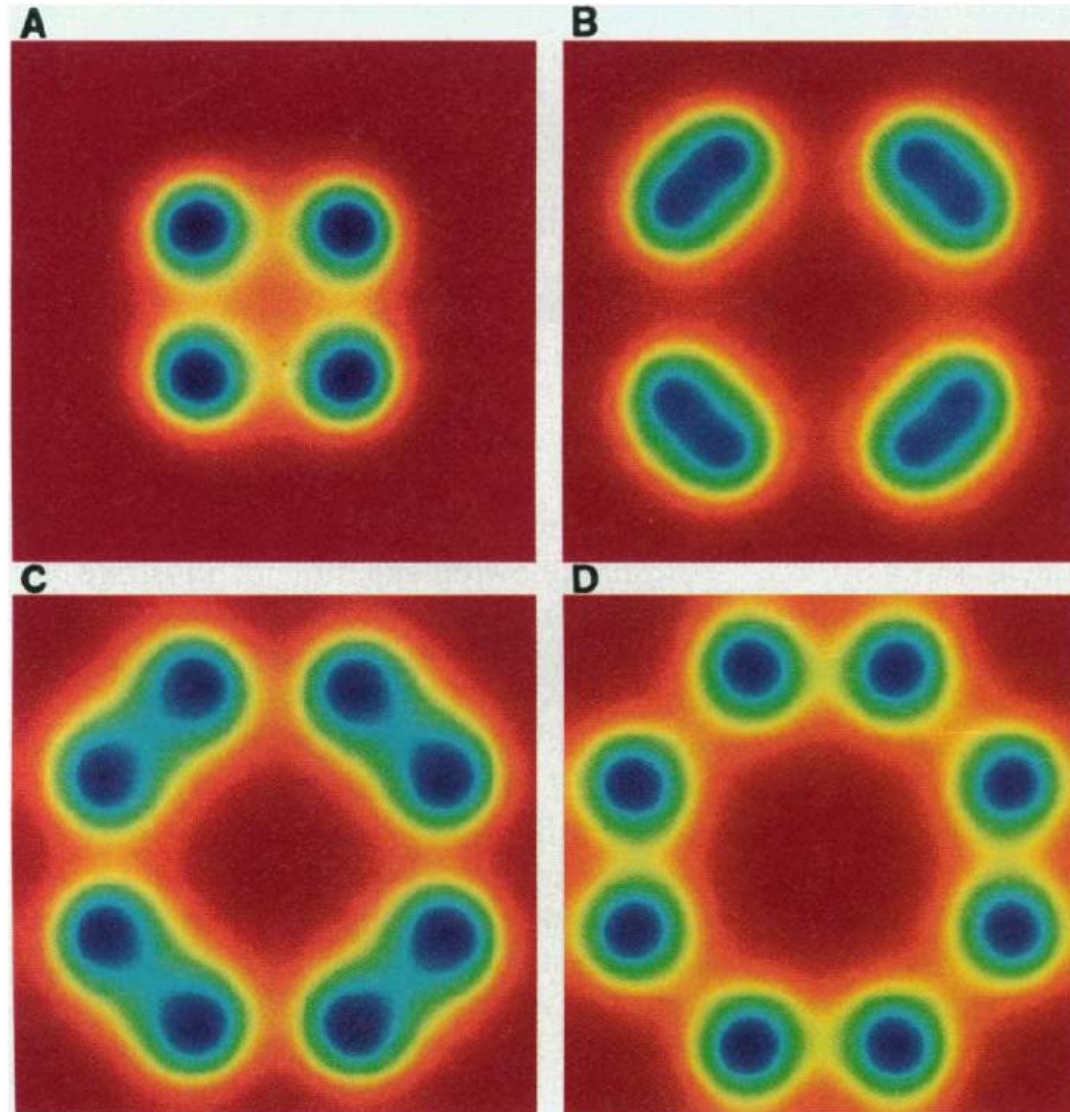


- A trivial steady-state solution ($U = 1, V = 0$) exists and is stable for all F and k
- Within the region shown there are two stable steady states
- Outside, a stable periodic solution is found for $k < 0.035$

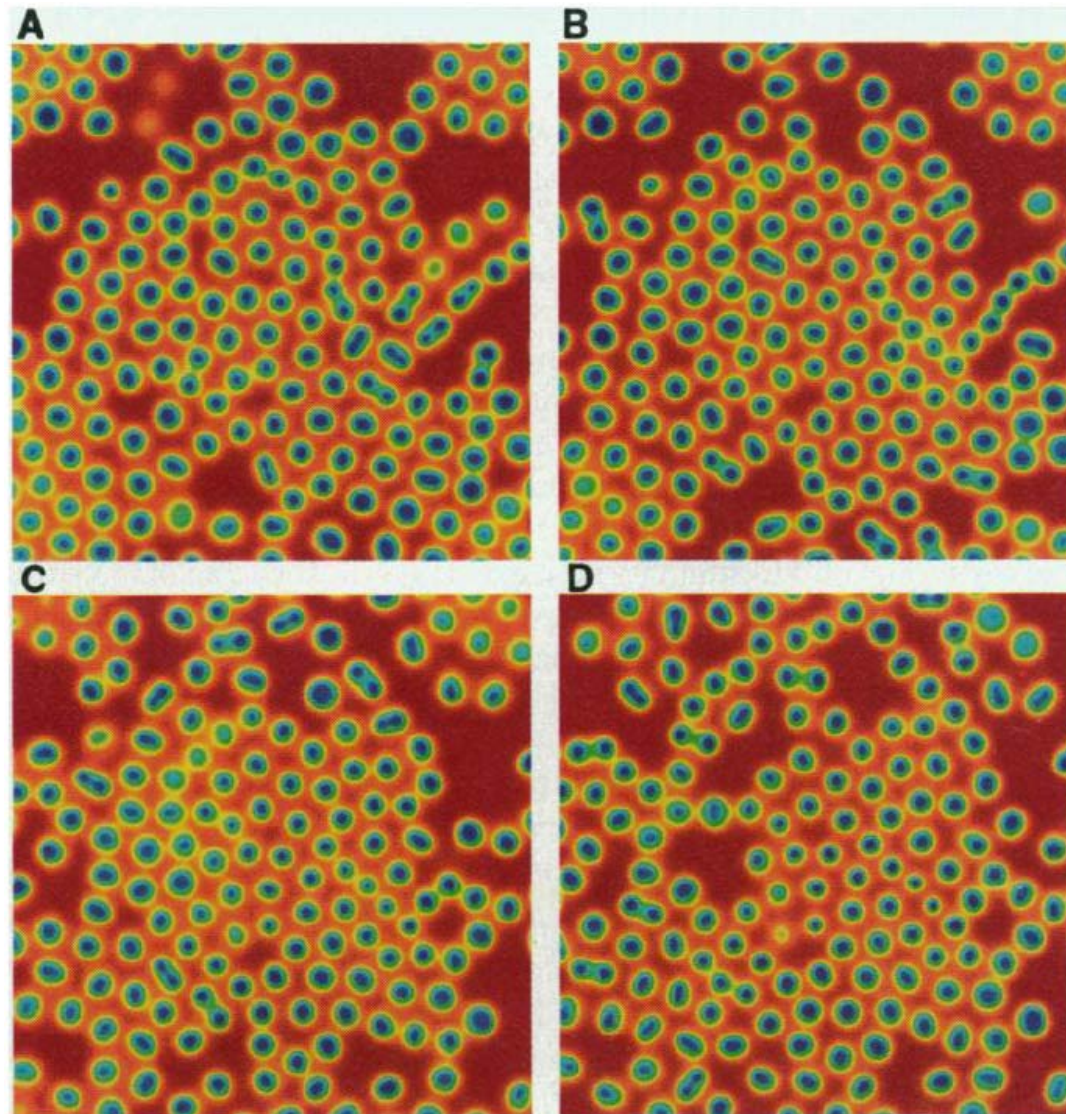
α  β  γ  δ  ϵ  ζ  η  θ  ι  κ  λ  μ 



Cell Division



Forming and Filling of Holes





Summary

- Simple rules lead to complex behavior
- More complex rules can lead to the same behavior but with a more continuous set of states
- Tweak parameters to obtain different patterns from the same rules
- Use different sets of rules in succession
- Achieve varying levels of stability in various regions of parameter space