Paper Summary: Exploring Complex Networks
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This paper explained about topologies of networks and their effects for both regular networks and complex networks which the authors have focus on dynamic system.

In order to study networks' topologies and their effects, several key-complications must be addressed, such as Structure Complexity, Network Evolution, Connection Diversity, Dynamical Complexity, Node Diversity, and Meta-Complication. These complications prevent researchers or scientists to have great understanding of networks' behaviors. To be able to effectively study networks' behaviors, some complications must be omitted while others can be deeper attacked. Therefore, the researchers would have great understanding on how networks' topologies affecting the networks' behaviors.

In generic dynamical networks, behaviors or final states of systems can be one of these three states, Stable fixed point, a limit cycle, and chaotic. The stable fixed point, particles in networks or systems will stop at some particular level. The limit cycle, the particles will move in the same patterns (cycle in this paper). And the chaotic, the movement patterns of particles will not repeat themselves.

For example, in case of dynamic system when a particle was isolated from others, it would be more active. If the particle has stable fixed point coupled together and doesn't have any interference, the result of this system is likely to be locked into some static patterns. If each particle has a chaotic attractor, the result of system might be chaos or synchrony. In an identical chaotic system, particles would likely be synchronized to others. For other networks to have synchronized systems, the coupling strength of particles can't be too weak or too strong.

In case of each particle has a stable limit cycle, the result can be synchronized or has some patterns which rely on the symmetry of the network. If they are fully connected, the result of this kind of network will be synchronized state. For systems which have non-identical particles with some degree of coupling, if the coupling is too weak compared to it own moving pattern, each particle will move with it own way. On the other hand, if the coupling is strong enough and exceeds some threshold, the particles will start to synchronize their movement. If the coupling strength is kept increasing, at some point all particles will move in synchrony.

The above behaviors have been tested under regular network, such as ring or fully connected networks which their connections or number of nodes are not changed overtime. In contrast, complex networks, connections, number of connection, and number of nodes in systems are subject to change at anytime, such as random graphs, small-world networks, and scale-free networks.

The random graphs can be created by randomly pick two nodes and connect these two nodes together. The process can be repeated to some satisfaction. If the number of connections (links) is too low (less than half of number of nodes), there will be several separated networks in the inspected system. If the number of links is about half of the number of nodes, the system will start forming one big network. When the number of links is kept increasing, the average shortest path between two nodes is about log(k); where 'k' is number of links. Molloy and Reed have proposed a formula to describe the network's fraction, Q = $\sum_{k=1}^{\infty} p_k\, k(k-2)$; where $p_k = d_k/n$, d_k is a node that has k links, and n is the number of nodes in the network. From the formula, if Q is less than zero, the system consists of many unconnected networks. The system will have less number of unconnected networks as the Q is close to zero, and from a big network when Q is greater than zero.
The Small-World networks can be created by rewiring or adding links between nodes with some probability to form short cut between nodes. The result of the network will be more clustering and has shorter shortest-path.

For the scale-free networks, the node can be added to the network with preferential attachment. The added node will be most likely attached to the existing node that has a high degree of links. Therefore the degree of distributions is followed a power law. This type of a network is claimed to be robust because randomly removing nodes from the network does not have great effect to the integrity of the network. On the other hand, the integrity of a network can be easily destroyed by removing a small number of high degree nodes (hub). Aiello, Chung, and Lu have applied Q-formula, $Q = \sum_{k=1}^{\infty} p_k k(k-2)$, to this type of network; where p_k is similar to $k^{-\gamma}$. The result of Q implies if $\gamma < 3.47$, there will be one big network in the system. And if $\gamma < 1.0$, there are many high-degree hubs to form one big-network

There is another type of graph, mentioned in the paper, is called bipartite graph. This graph has two types of nodes, and the connections between nodes cannot be connected to the same type of the node. The links can be established only between different types of nodes. This graph has been tested with a network of board of directors in the US companies. The result is satisfied and the prediction is perfectly matched with the actual distribution. But when testing with others, such as movie actors and biomedical authors, the result is half underestimating from the actual result. The graph loses some information about relationship between nodes.

This paper is kind of review of network topologies and their effects on the system. During the class we discussed a little bit about the network's complications, and network's characteristics.