

Spatial Communities 2

Evolutionary Prisoner's Dilemma

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Zero Sum Games

- 2 Children Cutting a Cake
 - One cuts the cake, the other chooses his piece
- Flipping Pennies
 - 2 people place pennies on a table. If both have the same side showing, player 1 takes both, else player 2 takes both.
- Nash Equilibrium: the point in the strategy space that corresponds to two stable strategies for the players
 - Cake-cutting: always cut the cake in half
 - Flipping pennies: randomly place your penny

Non-Zero Sum and Dilemmas

- In order for a dilemma to exist, players must have a possible reason for both cooperating and defecting.
- $(CC > CD) \wedge (DC > DD) \wedge ((DC > CC) \vee (DD > CD)) \wedge (CC > DD)$
- Three types of games meet these criteria: Chicken, Stag Hunt, and the Prisoner's Dilemma

Chicken

- Two cars drive on a collision course; the first one to swerve loses.
 - C: swerve
 - D: don't swerve
- $DC > CC > CD > DD$

Stag Hunt

- You and a partner are hunting a stag, which requires two people to take down. During the hunt, each of you comes across a hare, which only takes one person to kill. You can trade the stag (which feeds 2) for the hare (which feeds 1)
 - C: help your fellow hunter kill the stag
 - D: abandon your partner and hunt the hare (presumably because you assume he'll do the same)
- $CC > DC > DD > CD$

Prisoner's Dilemma

- You and a partner in crime are arrested. The DA offers you a deal: If you turn state's evidence against your partner, you live a life of luxury while your partner rots in prison. If both confess, then you both go to prison, but with a reduced sentence. If you both keep your mouth shut, you can be home in an hour.
 - C: Keep quiet
 - D: Rat out your partner
- $DC > CC > DD > CD$

Iterated Prisoner's Dilemma

- More than one round
- Complete knowledge of own and competitor's previous moves
- Always Defect
 - Impossible to win against it
 - Impossible to exploit
 - Can endlessly exploit forgiving strategies, but will suffer when playing unforgiving ones (like itself)

IPD

- Always Cooperate
 - Can be endlessly exploited
- Randomly Cooperate or Defect
 - Somewhat exploitative, somewhat resistant to exploitation
 - Does so-so when played against self.

Average Scores

- All-C: 1.5
- Rand: 2.16
- All-D: 3.0
- All-D optimizes local fitness at expense of global fitness, and All-C is vice-versa
- Both All-* are therefore bad, and Rand is unsatisfying.
- Perhaps because it does not take advantage of memory?

Enter the Winner

- Tit for Tat (TFT)
 - Offer to cooperate in the first round
 - In every subsequent round, do what your opponent did last round.
- Won the first round, then won a second round against programs designed specifically to beat TFT
- May not be “best,” but works in a wide variety of settings

TFT

- Seek win-win situations and don't get greedy for higher payoffs
- Never be the first to defect
- Elicit cooperation by rewarding cooperation and punishing defection (then forgiving)
- Stay simple
- (NetLogo: 2-person Iterated Prisoner's Dilemma)

How can a successful strategy evolve?

- It must be initially viable
- It must be robust to varied environments (which All-D and All-C are not, because they rely on other “nice” strategies)
- It must be resistant to invasion from mutated strategies
 - Evolutionary stable strategy: a strategy that can suppress individual invaders, like All-D, and unlike All-C

Axelrod & Hamilton

- What pressures led to the evolution of cooperation in living things?
 - Kinship theory: though your own genes may die, the copies present in your relatives may live on
 - Reciprocation theory: you gain specific advantages by giving of yourself to others, e.g., symbioses
- However, altruism has been observed in groups with low relatedness, and cheating has been observed in symbiotic relationships.

The Cooperation Model

- Cooperation is based on the probability that two agents will interact at some point in the future.
- Other agents in the environment can affect the strategy of cooperation
- Can scale down to the microbial level to speculatively describe behavior of diseases

The Evolutionary Inevitability of All-D

- An All-D strategy is the outcome of inevitable evolutionary trends through mutation and natural selection.
- If the fitness is the PD payoff, and interactions are random and not repeated, any population will evolve to be defectors; no single strategy can overcome it (evolutionarily stable).
- However, the same two individuals may meet more than once, making it beneficial for them to cooperate

Biological Basis

- Bacteria can play games, in that they are sensitive to changes in the environment and can take different actions based on those changes. Judgments on likelihood of “meeting” another agent again may be inherited
- Primates have more complex memory and reasoning, allowing more information to go into choosing current actions, better estimation of probability of future interactions, and better ability to distinguish between individuals

The evolution of Cooperation

- Can be conceptualized in terms of three main points
 - Robustness
 - Stability
 - Initial Viability

Robustness

- Exemplified by Tit-For-Tat
 - Never the first to defect
 - Provocable into retaliation by defection of the other
 - Forgiving after only one act of retaliation

Stability

- If w is the probability that two strategies will interact at some point in the future, it can be shown that NO strategy can invade TFT as long as:
 - $w \geq (T - R) / (T - P)$
 - $w \geq (T - R) / (R - S)$
- Where T is the payoff for DC, R is the payoff for CC, P is the payoff for DD, and S is the payoff for CD

Initial Viability - Relatedness

- So TFT is evolutionarily stable, but so is All-D. So how could a particular strategy evolve at all in the presence of another?
 - Goes back to kinship theory—sacrifice of fitness for a relative's benefit, or for mutual benefit through sharing of fitness.
 - Once these sharing genes are present, cooperation is based on perceived relatedness.
 - Relatedness due to promiscuous fatherhood
 - Ill-defined group margins
 - Reciprocation of cooperation

Initial Viability - Clustering

- If a cluster of agents using TFT is placed in an All-D environment, they can gain more by cooperating with each other than the All-D agents can by interacting with the TFT's, and thus gain a foothold.
- However, it is impossible for a cluster to take over TFT (or another “nice,” stable strategy)
 - Clusters of nice strategies will gain as much as or less than TFT
 - Small clusters of mean strategies will always gain less

Applications

- Immediate, drastic retaliation: fig wasps
- Employing a fixed place of meeting: aquatic parasitophages.
- Collocation: ants and honeybees
- Stable territories: birds and identification songs
- Recognition of individuals allows for more stable cooperation strategies: humans
- Exploitation of reduced probability of future meeting: lymphoma and malaria, gut bacteria

Ecology and Spatiality

- We see how the environment can affect the strategy, but how can the strategy affect the environment? (Ecology)
- Ecological model: each strategy is an organism, of which the environment can support only a limited amount.
- Thus, a particular strategy's population can be expressed as a proportion of the overall population.

Ecological Iterated Prisoner's Dilemma

- Keep track of the populations and the scores, as well as a lookup table (R) that holds the relative scores (R_{ij}) of two strategies (i and j) played against each other for some fixed number of iterations
- The population of a strategy at the next time step is proportional to its current population times its score
- The score of a strategy at a given time step is the sum of its relative scores weighted by the populations of the strategies with which it is competing

Ecological Model - Details

- Payoff Matrix:
 - $DC = 5$, $CC = 3$, $DD = 1$, $CD = 0$
- Initial populations:
 - All-C: 60%
 - All-D: 10%
 - TFT: 10%
 - Rand: 20%
- Each time step consists of 200 PD iterations

Enter the Pavlovian

- Win:Stay, Lose:Shift
 - Cooperate at first and until opponent defects
 - Switch to defection until opponent defects again
 - Return to first step
- Makes a reasonable showing under normal circumstances
- Really shines when noise is introduced into the communications
 - Can correct itself after accidental defection, unlike TFT
 - Can take advantage of All-C after realizing it can get away with it

Spatial Model

- “Assuming that ecosystem members will interact with each other with frequency that is directly proportional to population levels is a lot like assuming that every one in a city will talk with every one person in equal proportions.”
- Here, we introduce the concept of spatial division (a citizen will normally only talk to his nearest neighbors).

Spatial Prisoner's Dilemma

- Hold IPD contests between immediate neighbors on a toroidal grid.
- At each time step, each cell computes an overall score for itself.
- Each cell adopts the strategy used by the neighbor with the highest score.
- Ties go to the current strategy.

SPD

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- A small cluster of cooperating agents can prosper in a hostile environment
 - Parasitic agents can exist only in limited numbers

Nowak & May

- Modify the payoff matrix such that:
 - Mutual cooperators score 1
 - Mutual defectors score 0
 - Defection against cooperation scores b (s.t. $b > 1$)
 - Cooperation against defection scores 0
- Different behavior in the population becomes evident as b changes.

Spatial Prisoner's Dilemma

- If $b > 1.8$, D clusters will grow
- If $b < 1.8$, Larger D clusters will shrink
- If $b > 2$, C clusters won't grow or will shrink
- If $b < 2$, small C clusters will grow
- Thus, if $1.8 < b < 2$, we get interesting, chaotic behavior as clusters of both C and D grow, move, and collide
- Proportion of C present approaches .318
- (NetLogo: Evolutionary PD)

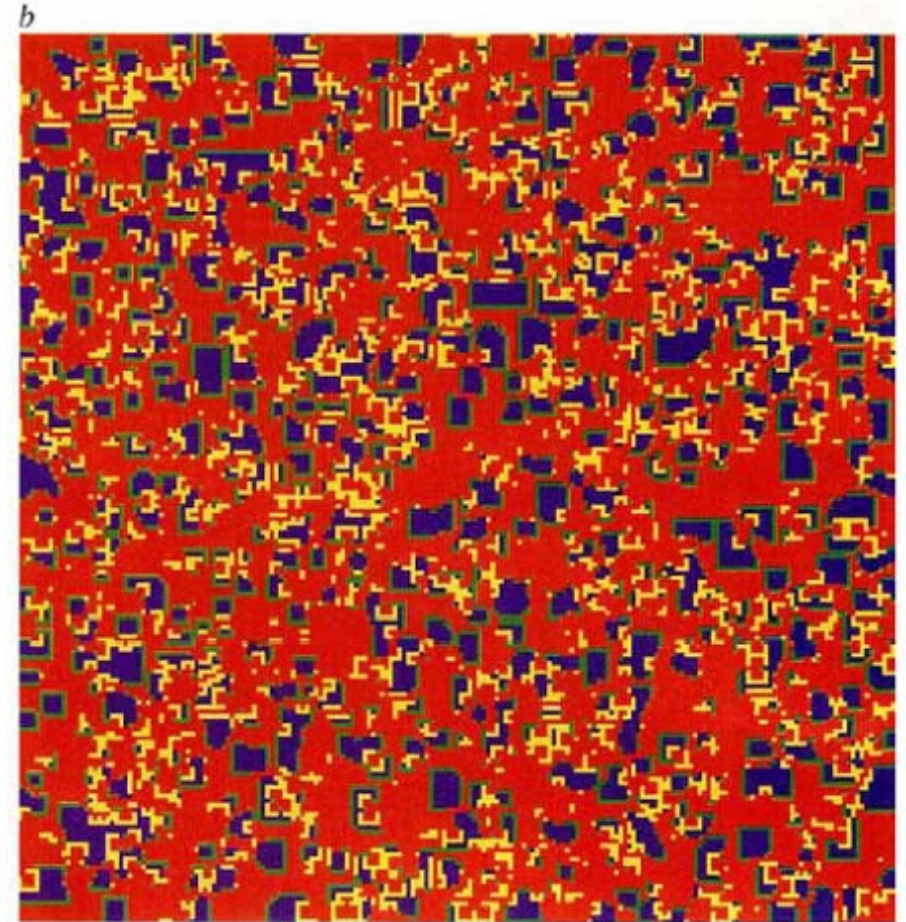
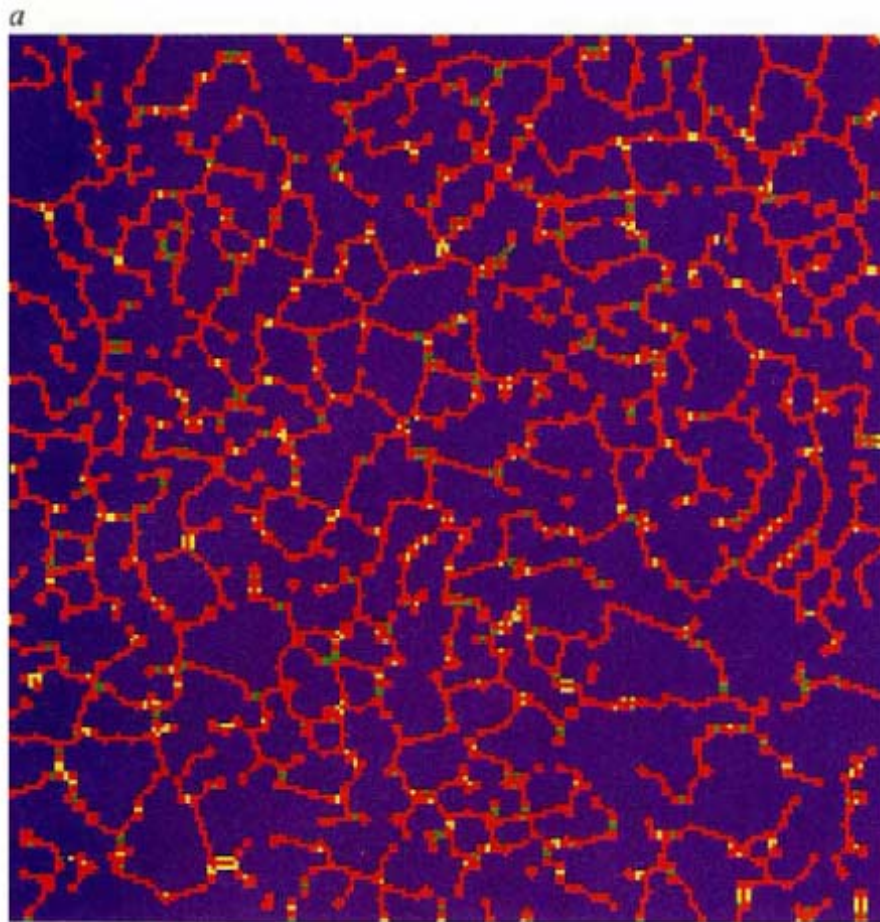
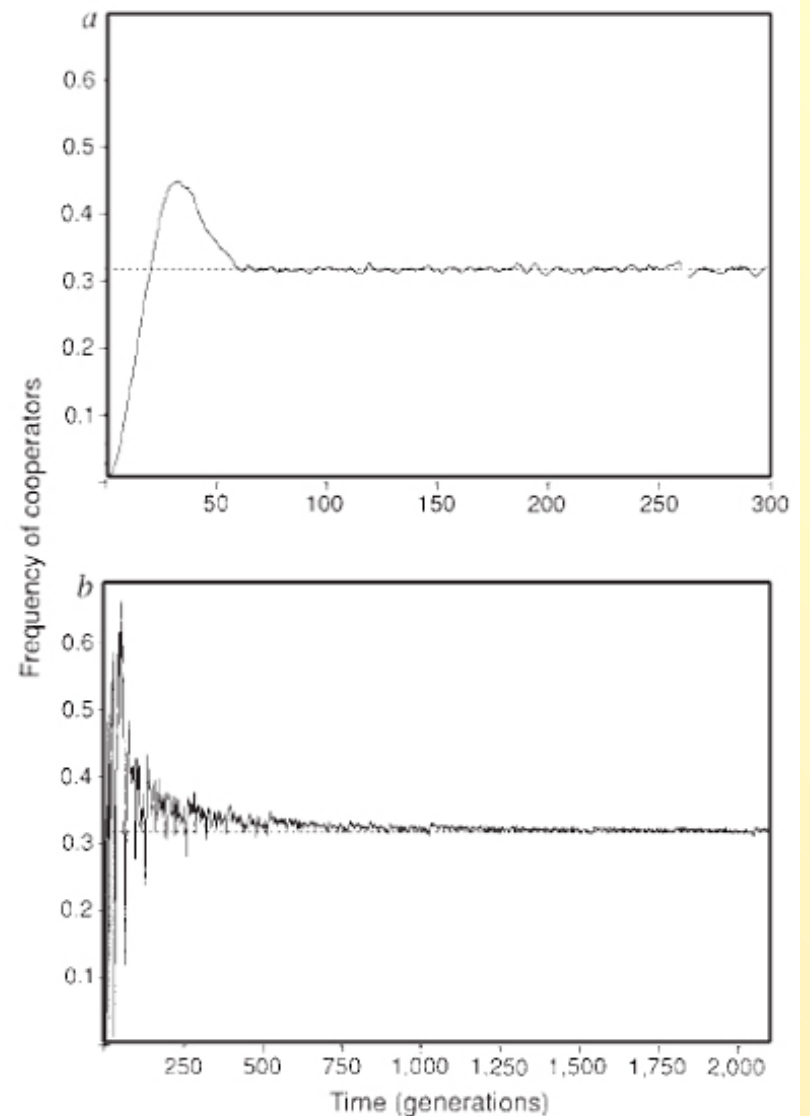


FIG. 1 The spatial Prisoners' Dilemma can generate a large variety of qualitatively different patterns, depending on the magnitude of the parameter, b , which represents the advantage for defectors. This figure shows two examples. Both simulations are performed on a 200×200 square lattice with fixed boundary conditions, and start with the same random initial configuration with 10% defectors (and 90% cooperators). The asymptotic pattern after 200 generations is shown. The colour coding is as follows: blue represents a cooperator (C) that was already a C in the preceding generation; red is a defector (D) following a D; yellow a D following a C; green a C following a D. *a*, An irregular, but static pattern (mainly of interlaced

networks) emerges if $1.75 < b < 1.8$. The equilibrium frequency of C depends on the initial conditions, but is usually between 0.7 and 0.95. For lower b values (provided $b > \frac{9}{10}$), D persists as line fragments less connected than shown here, or as scattered small oscillators ('D-blinkers'). *b*, Spatial chaos characterizes the region $1.8 < b < 2$. The large proportion of yellow and green indicates many changes from one generation to the next. Here, as outlined in the text, 2×2 or bigger C clusters can invade D regions, and vice versa. C and D coexist indefinitely in a chaotically shifting balance, with the frequency of C being (almost) completely independent of the initial conditions at ~ 0.318 .

FIG. 2 The frequency of cooperators in simulations with random or symmetrical initial conditions, within the interesting region $1.8 < b < 2$. *a*, The frequency of cooperators, $f_C(t)$, for 300 generations, starting with a random initial configuration of $f_C(0) = 0.6$. The simulation is performed on a 400×400 square lattice with fixed boundary conditions, and each player interacts with 9 neighbours (including self). The dashed line represents $f_C = 12 \log 2 - 8$ (see *b*). *b*, The frequency of C within the dynamic fractal generated by a single D invading an infinite array of C. At generation t , the width for the growing D structure is $2t + 1$, and the frequency of C, $f_C(t)$ within the square of size $(2t + 1)^2$ centred on the initial D site is shown as a function of t . This curve has interesting properties, which can be partly understood by referring to the geometry of the D structure. The D structures are closed-boundary squares in generations that are powers of 2, $t = 2^n$; hence $f_C(t)$ has minima at generations that are powers of 2. These squares now expand at the corners and erode along the sides, returning to square shape after another doubling of total generations. On this basis, a crude approximation suggests that, i time steps going from t to $2t$, there will be roughly $4(2i)(2t + 1 - 2i)$ C sites within the D structure of size $(2t + 1 + 2i)^2$; hence the asymptotic C fraction, f_C , for very large such symmetrical patterns is $f_C = 4 \int_0^1 s(1-s)/(1+s)^2 ds = 12 \log 2 - 8 = 0.318 \dots$. As discussed in the text, this approximation agrees with the numerical results surprisingly well in both parts *a* and *b* of the figure.



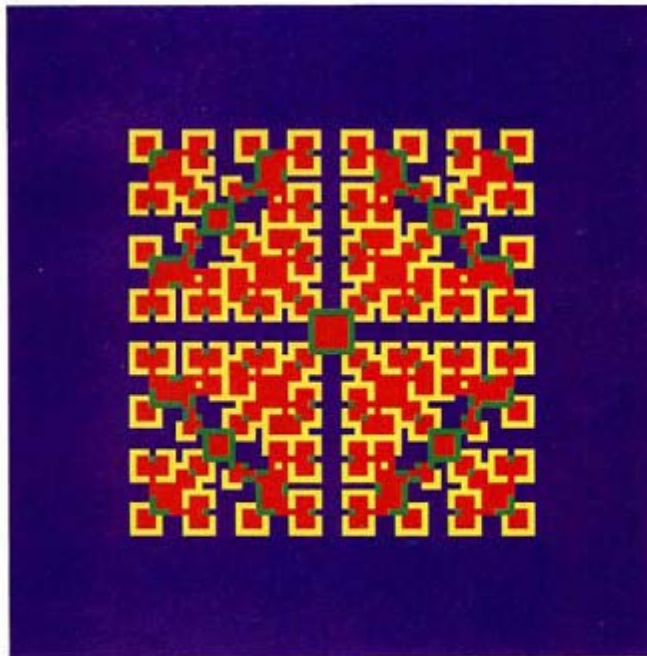
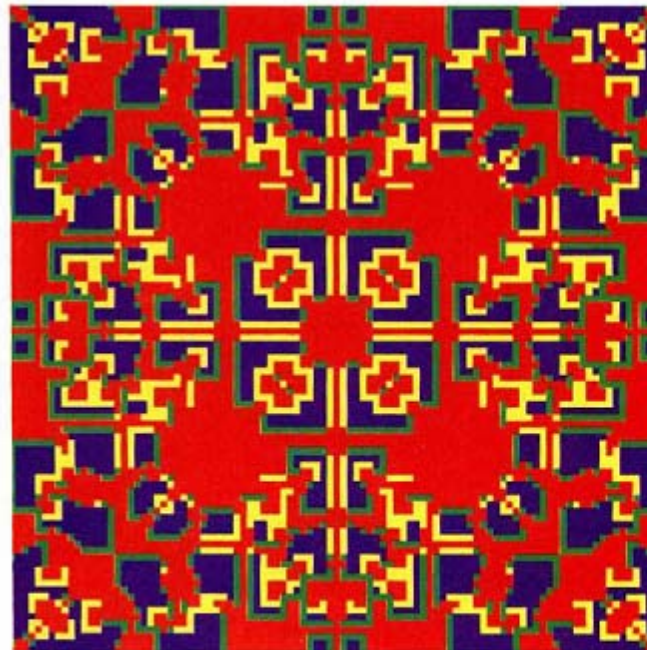
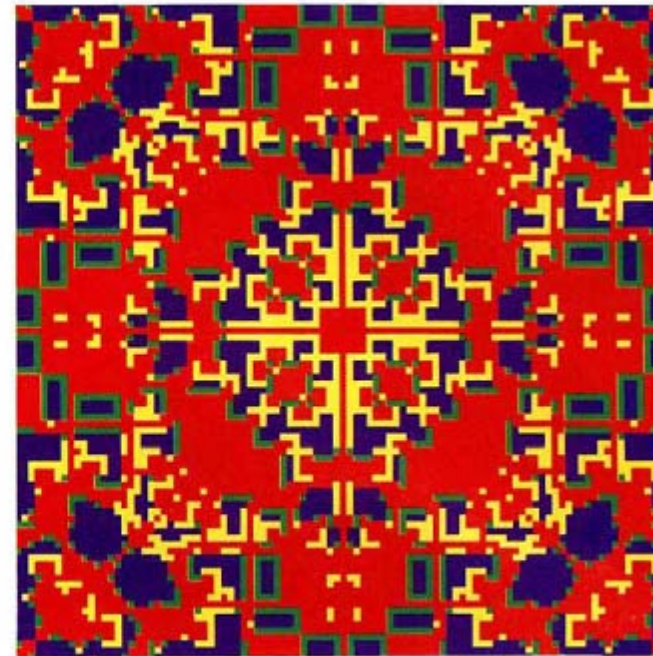
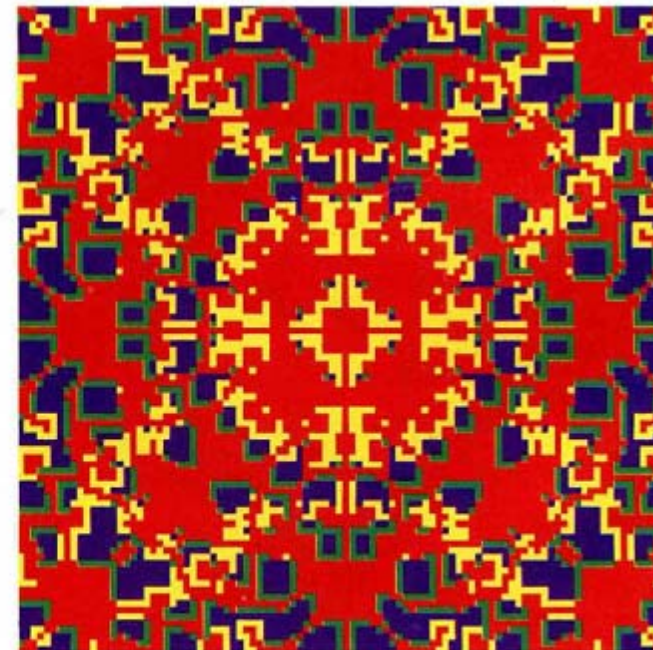
a*b**c**d*

FIG. 3 Spatial games can generate an 'evolutionary kaleidoscope'. This simulation is started with a single D at the centre of a 99×99 square-lattice world of C with fixed boundary conditions. Again $1.8 < b < 2$. This generates an (almost) infinite sequence of different patterns. The initial symmetry is

always maintained, because the rules of the game are symmetrical. The frequency of C oscillates (chaotically) around a time average of $12 \log 2/8$ (of course). *a*, Generation $t=30$; *b*, $t=217$; *c*, $t=219$; *d*, $t=221$.

Conclusions

- These ideas appear in natural systems (parasite-cleaning fish, WWI soldiers, international trade, etc.).
- If there is no chance of meeting again, defection is the better strategy, and cooperation becomes better as probability of future encounters increases
- The best strategies involve both cooperation and defection, based on what has gone before and what is still to come.