CS 790R Seminar

Spatial Communities 1 – Spatial Ecology

Milind Zirpe
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Department of Computer Science & Engineering
University of Nevada, Reno
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Discussion

- Flake (1998), Chapter 12
- NetLogo demos:
  - Wolf Sheep Predation (individual model)
  - Wolf Sheep Predation (docked) (individual and aggregate models)

Introduction

- Producer-Consumer interactions
- Simple Lotka-Volterra system
- Generalized Lotka-Volterra system
- Individual based system
- Conclusion

Producer-Consumer interactions

- Couple of techniques for modeling population of species:
  - Simple aggregate simulation (model each species as a simple function of the other – species-eye view of the world).
  - Individual based simulation (model each individual of the population separately and simulate each one simultaneously – animal-eye view of the world).

Producer-Consumer interactions

- Most systems undergo a stabilizing process which tries to take them to equilibrium (e.g. population dynamics, heat regulation in mammals).
- Stabilization is easy when the state of an environment is mostly independent of the state of an individual (e.g.: body temperature in mammals does not have coupling effect with surrounding environment).

Producer-Consumer interactions

- Predator-prey systems are tightly coupled (Change in one's state has an effect on the other's state).
- Everything is connected to everything else in an endless web interactions. A small ripple in one location may be transformed into a tidal wave elsewhere (*Chaos theory*: the *butterfly effect*).

Simple Lotka-Volterra system

- Introduced independently by Alfred J. Lotka and Vito Volterra around 1920 (chemical reaction) and 1926 (predator-prey relationship) respectively.
- Two species with one (sharks) preying on another (small fish) leading to predator-prey coupled oscillating system.
- No other predators (hunting by humans, parasites, etc…).

Simple Lotka-Volterra system

- Couple of differential equations:

$$\frac{dF}{dt} = F(a - bS) \quad \text{and} \quad \frac{dS}{dt} = S(cF - d),$$

F: small fish population.

S: shark population.

a: reproduction rate of small fish.

b: proportional to the number of small fish that a shark can eat.

c: amount of energy that a small fish supplies to the consuming shark.

d: death rate of the sharks.

Simple Lotka-Volterra system

Observations:

- Each equation has *F* and *S* term - coupled system.
- In absence of predators, change in small fish population is: "*Fa*". (exponential growth).
- "*FS*" is the chance that a random shark will encounter a random small fish.
- Small fish population will decrease by "*bFS*" term.

Simple Lotka-Volterra system

Observations:

- Shark population will increase by an amount proportional to "cSF". (directly proportional to value of c).
- In absence of small fish (F=0), shark population will decay exponentially ($-Sd$).
- Shark population increases proportionally to small fish population but simultaneously decreases due to constant death rate.
- Fixed point of system at: $F=d/c$ and $S=a/b$. ($dF/dt=dS/dt=0$).

Simple Lotka-Volterra system

- Limit Cycles:

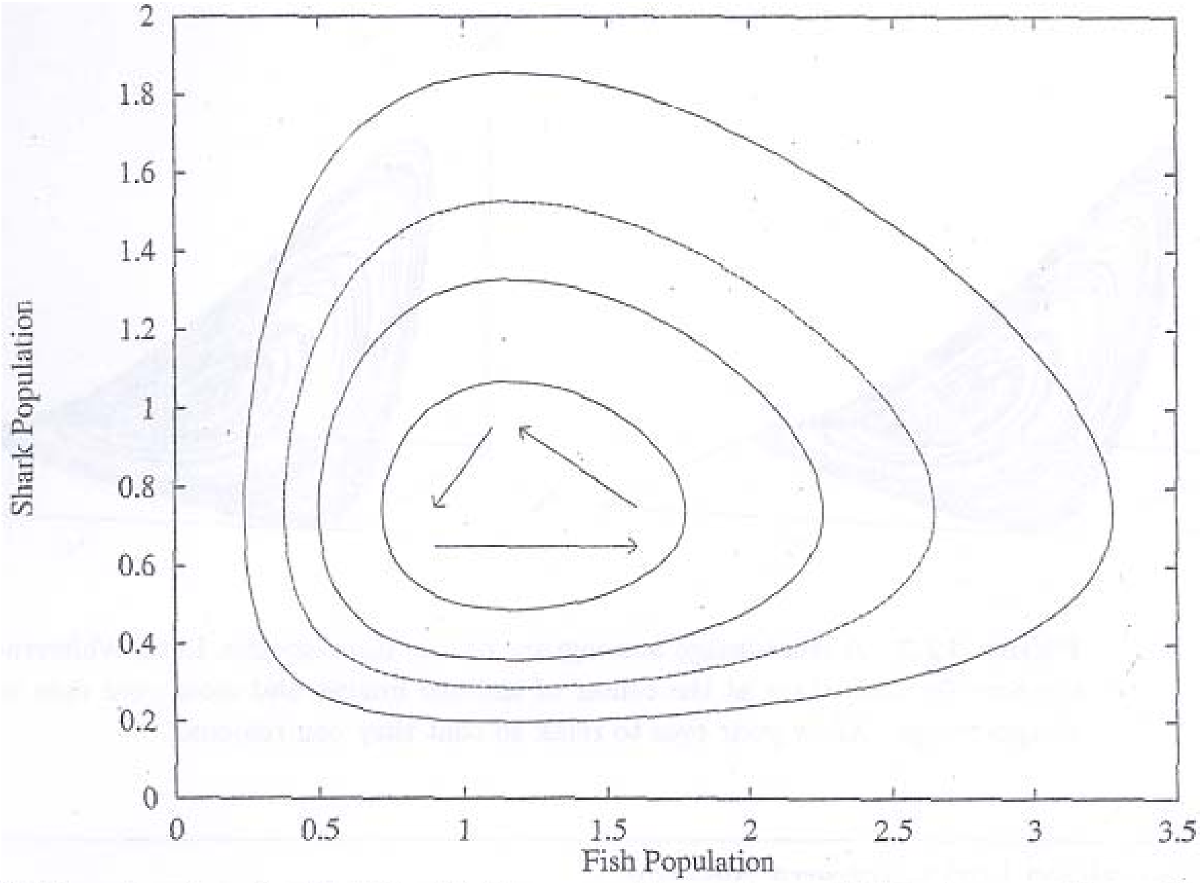


Figure 12.1 *A simple Lotka-Volterra attractor which shows four (out of an infinite number of possible) limit cycles.* The value of the four paramters are equal to 3.029850, 4.094132, 1.967217, and 2.295942, which yields a fixed point at 1.1671, 0.740047.

Simple Lotka-Volterra system

Observations:

- Infinite number of Limit cycles orbiting around the embedded fixed point.
- Change in either population forces system into different limit cycle.
- How to increase population level of small fish ?
  - Increase *a* ?   - No
  - Increase *d* or decrease *c* ?   - Yes

Generalized Lotka-Volterra system

- Three species predator-prey system (chaos in motion).
- In continuous systems, for something to be chaotic, it must never repeat itself, but it must return to a very similar state that it was at before.
- Scribbling on paper in 2-D, there will be line intersection (repeating) eventually.
- Hence, continuous chaos can exist in three or more dimensions.

Generalized Lotka-Volterra system

- System discovered by A. Arneodo, P. Coullet and C. Tresser.
- Differential equations for an *n*-species system:

$$\frac{dx_i}{dt} = x_i \sum_{j=1}^{n} A_{ij}(1 - x_j),$$

x_i represents the i^{th} species,

A_{ij} represents the effect that species *j* have on *i* species (similar to parameters in last model).

$$A = \begin{matrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{matrix} = \begin{matrix} 0.5 & 0.5 & 1.0 \\ -0.5 & -0.1 & 0.1 \\ \alpha & 0.1 & 0.1 \end{matrix}$$

Whole system can be controlled by a single variable α (chaotic behavior at $\alpha = 1.5$).

Generalized Lotka-Volterra system

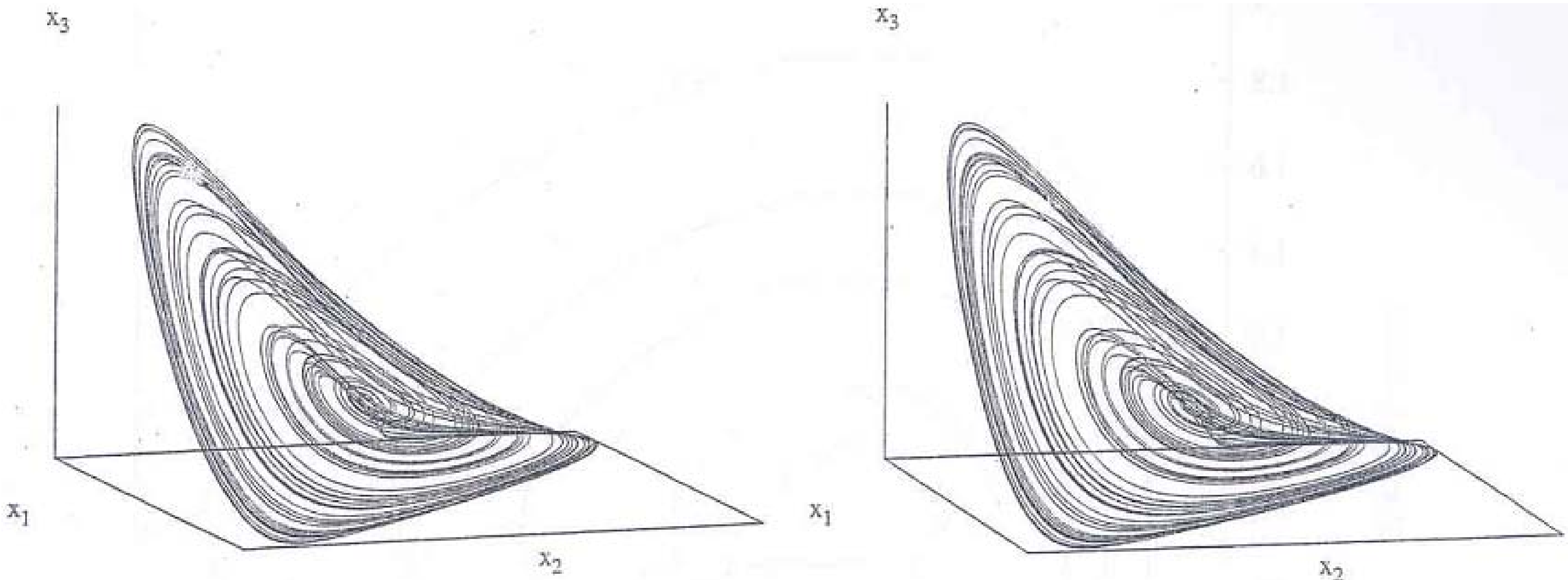


Figure 12.2 A dual-image stereogram of the three-species Lotka-Volterra chaotic attractor: To view, stare at the center of the two images and cross your eyes until the two images merge. Allow your eyes to relax so that they can refocus. $\alpha = 1.5$

Generalized Lotka-Volterra system

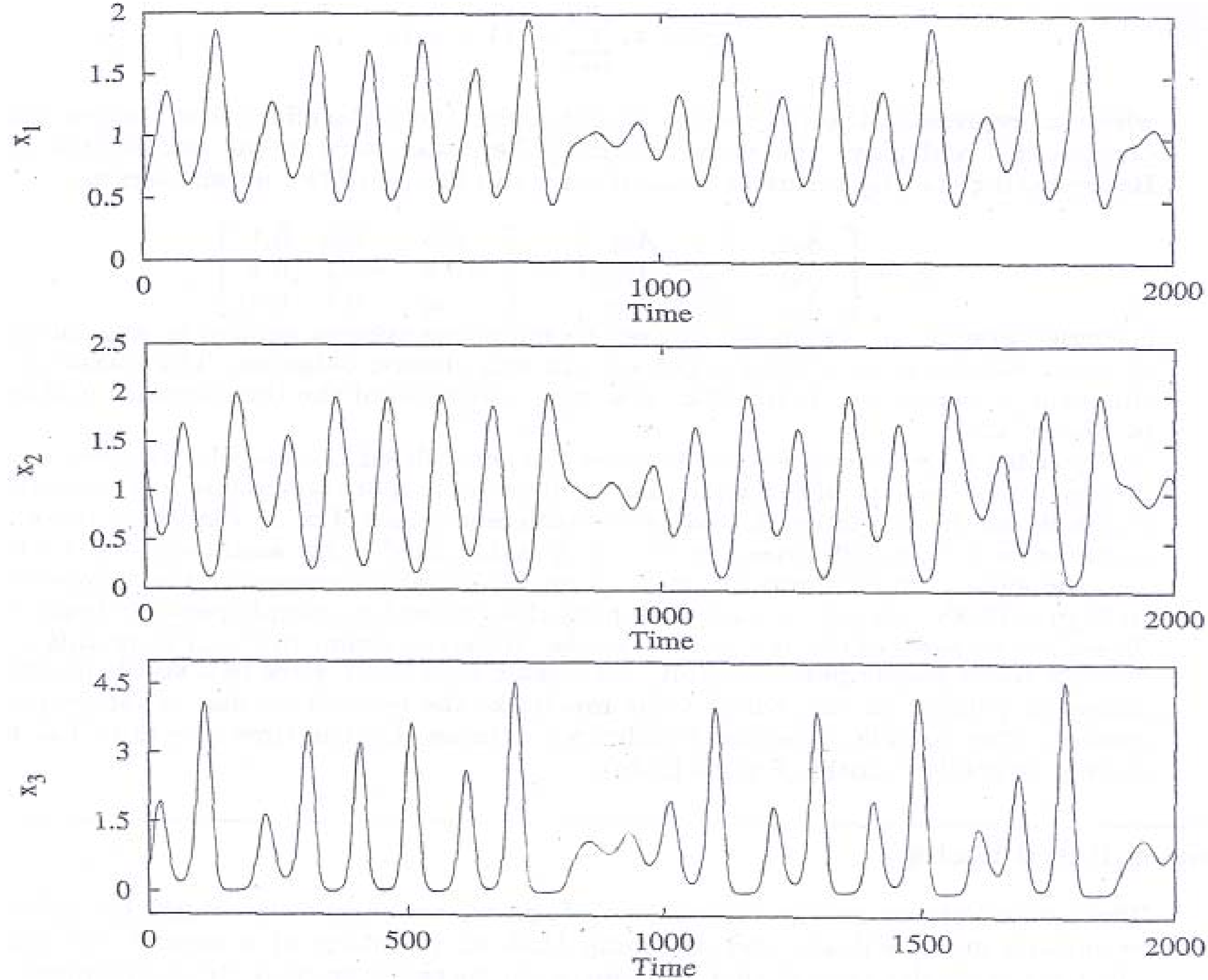
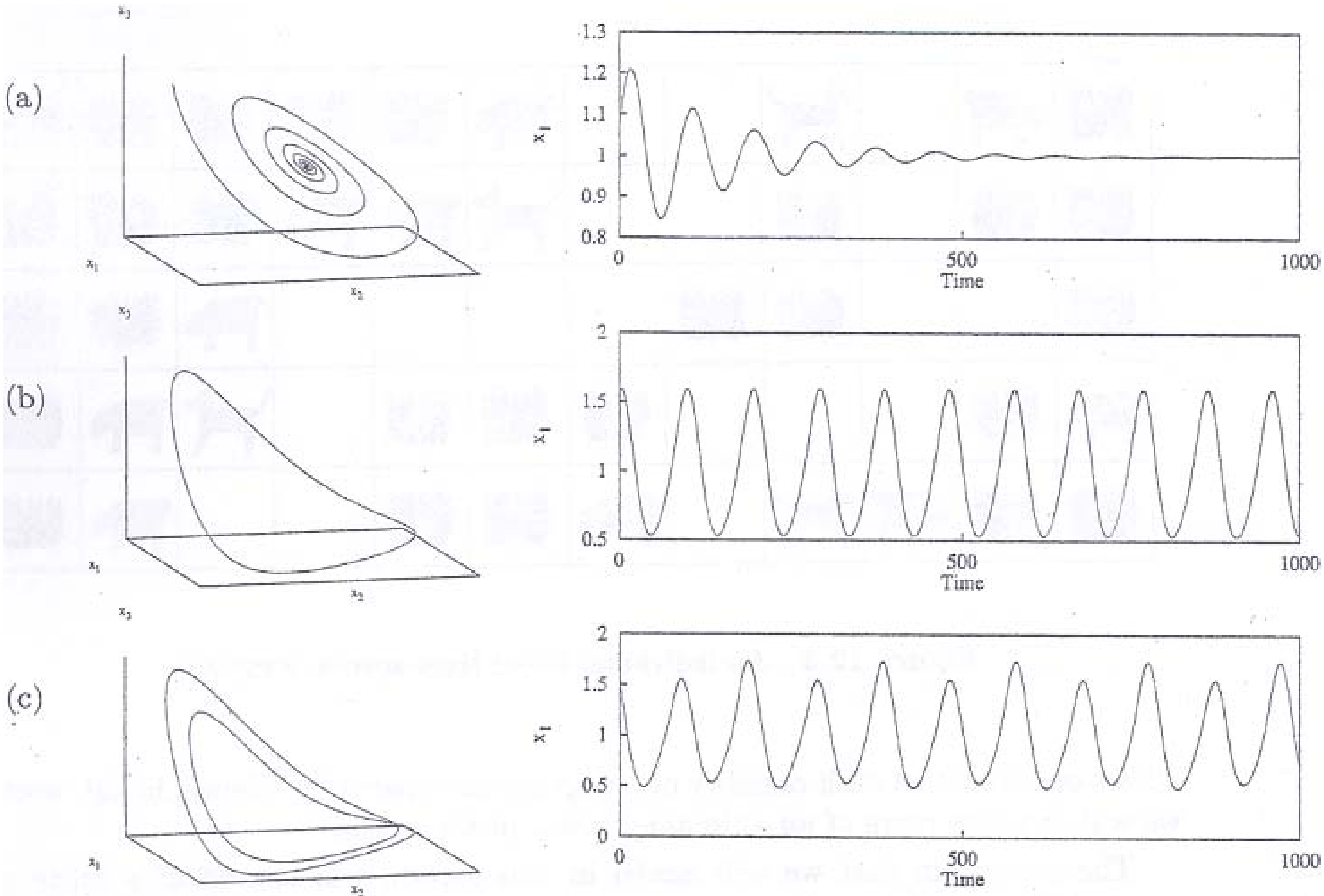


Figure 12.3 Population levels for the three-species Lotka-Volterra system

Generalized Lotka-Volterra system



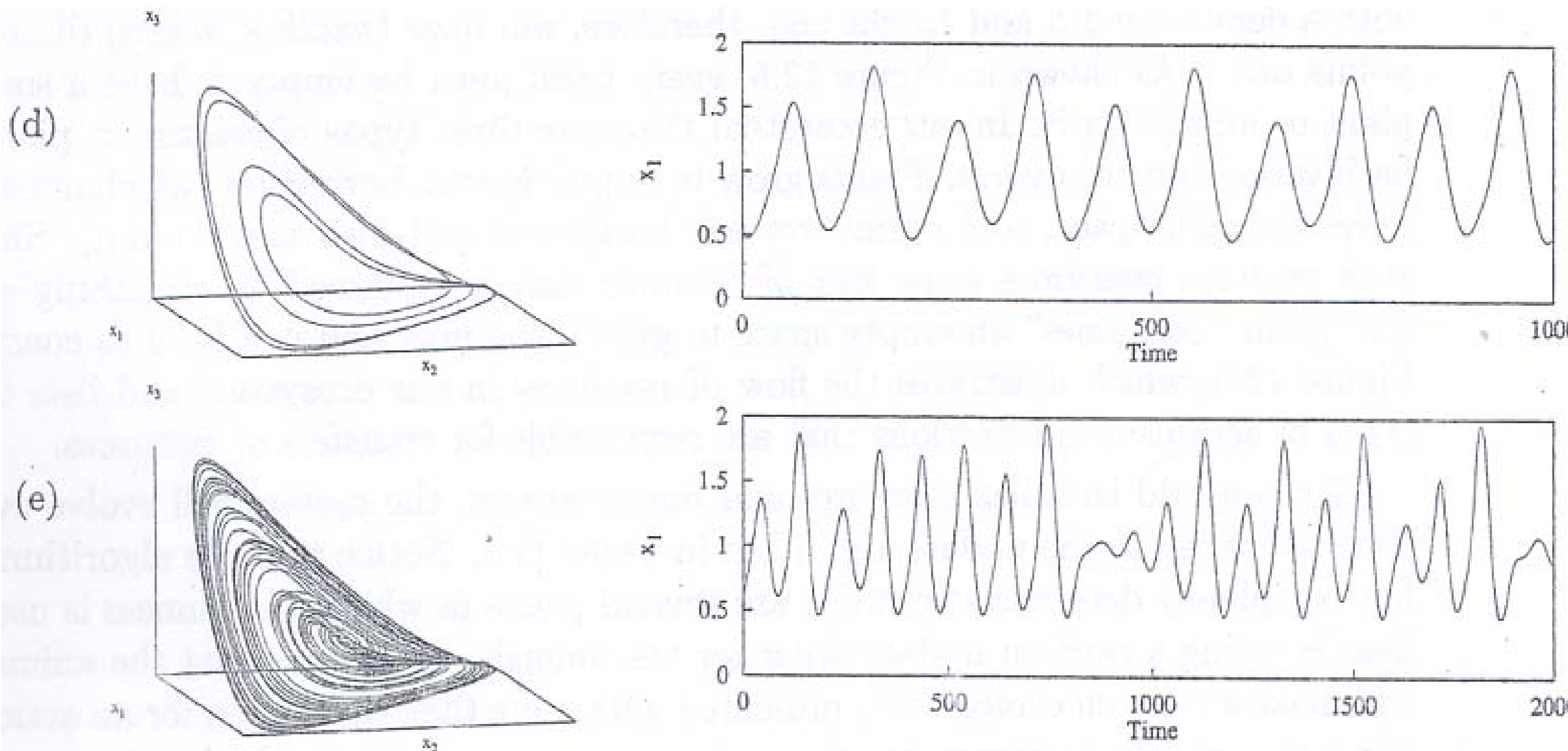
$\alpha = 0.75$

$\alpha = 1.2$

$\alpha = 1.32$

Figure 12.4 Period doublings in a three-species Lotka-Volterra system: phase space is on the left and x_1 is plotted on the right. (a) spiral fixed point, (b) simple periodic orbit, (c) period-2 orbit

Generalized Lotka-Volterra system



α = 1.387

α = 1.5

Figure 12.4 (d) period-4 orbit, (e) chaos

Individual based system

- Each individual of a species is modeled separately.
- Technique depends on a cellular automaton.
- Ecosystem consists of a finite grid (fixed width and height).
- Possible state of a grid-cell: empty, single plant or animal.
- Three types of things: plants, herbivores and carnivores.

Individual based system

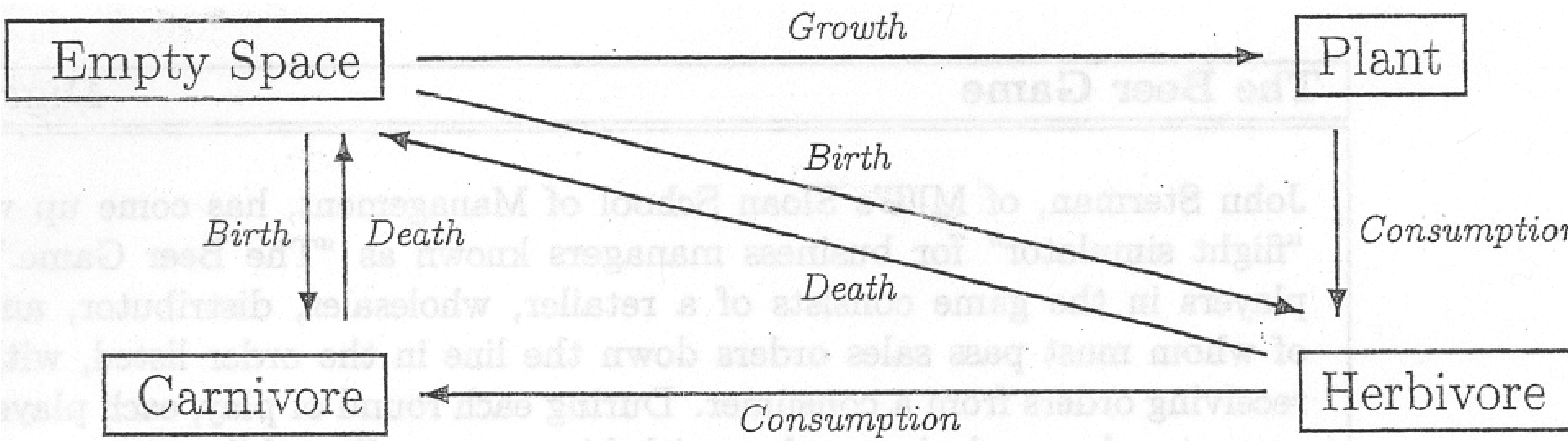


Figure 12.6 Flow of resources in the three-species individual-based ecosystem

- For every time step:
  - For every empty cell, e:
    - If e has three or more neighbors that are plants, then e will become a plant at the next time step (assuming it isn't trampled by a herbivore or carnivore).
  - For every herbivore, h (in random order):
    - Decrease energy reserves of h by a fixed amount.
    - If h has no more energy, then h dies and becomes an empty space.
    - Else, if there is a plant next to h, then h moves on top of the plant, eats it, and gains the plant's energy.
      - If h has sufficient energy reserves, then it will spawn a baby herbivore on the space that it just exited.
    - Else, h will move into a randomly selected empty space, if one exists, that is next to h's current location.
  - For every carnivore, c (in random order):
    - Decrease energy reserves of c by a fixed amount.
    - If c has no more energy, then c dies and becomes an empty space.
    - Else, if there is a herbivore next to c, then c moves on top of the herbivore, eats it, and gains the herbivore's energy.
      - If c has sufficient energy reserves, then it will spawn a baby carnivore on the space that it just exited.
    - Else, c will move into a randomly selected empty space that is next to c's current location. If there are no empty spaces, then c will move through plants.

Table 12.1 Update algorithm for individual-based ecological model

Individual based system

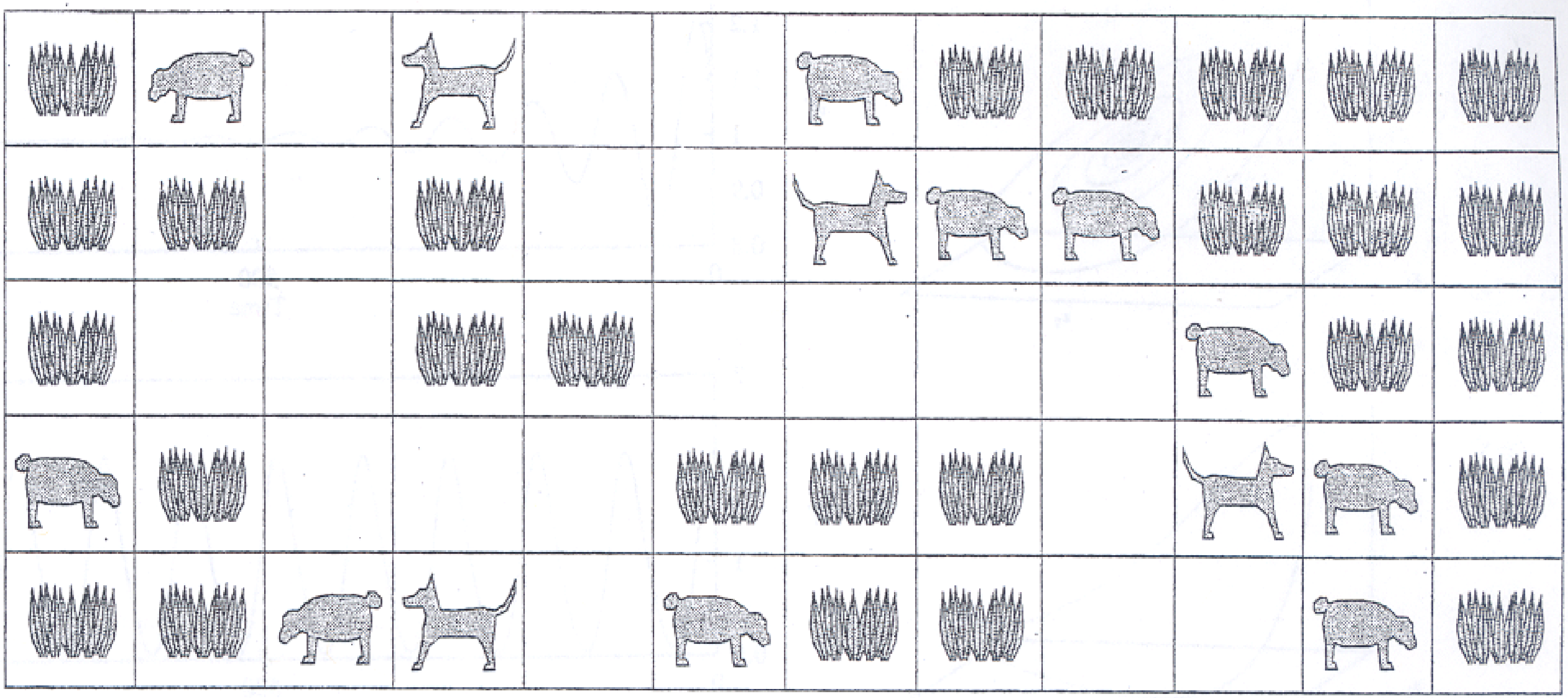


Figure 12.5 An individual-based three-species ecosystem

Individual based system

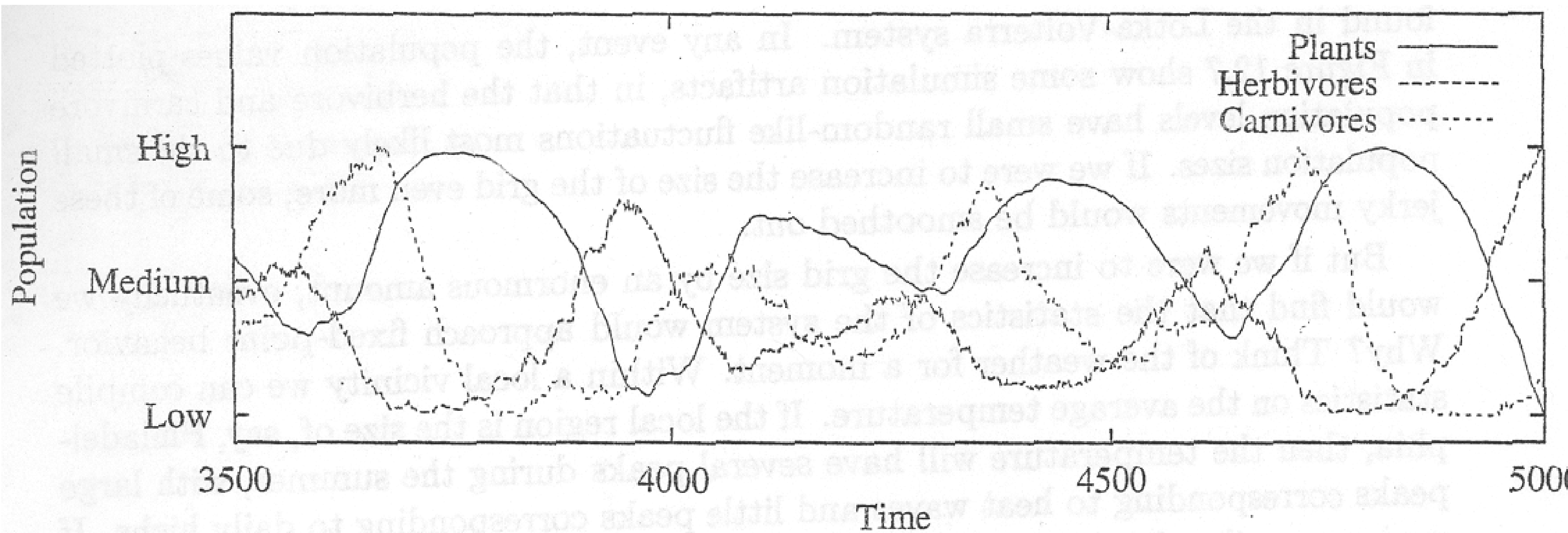


Figure 12.7 Population levels for all creatures, normalized for comparison

Individual based system

Observations:

- Number of legal states for individual-based ecosystem easily approaches astronomical numbers as grid size increases (say 1000 x 1000).
- Lotka-Volterra system uses three real numbers for its state, but still confined to 3-D space.
- State space of individual-based ecosystem can easily require thousands of dimensions.

Individual based system

Observations:

- Individual-based model (many subunits) is far more complicated than simpler Lotka-Volterra systems.
- Increasing the grid size by an enormous amount will lead the system to fixed-point behavior. (tiny ecosystems yielding randomness and enormous ecosystem yielding static behavior).
- Thus, the dynamics of the system collapse onto a lower-dimensional space.

Individual based system

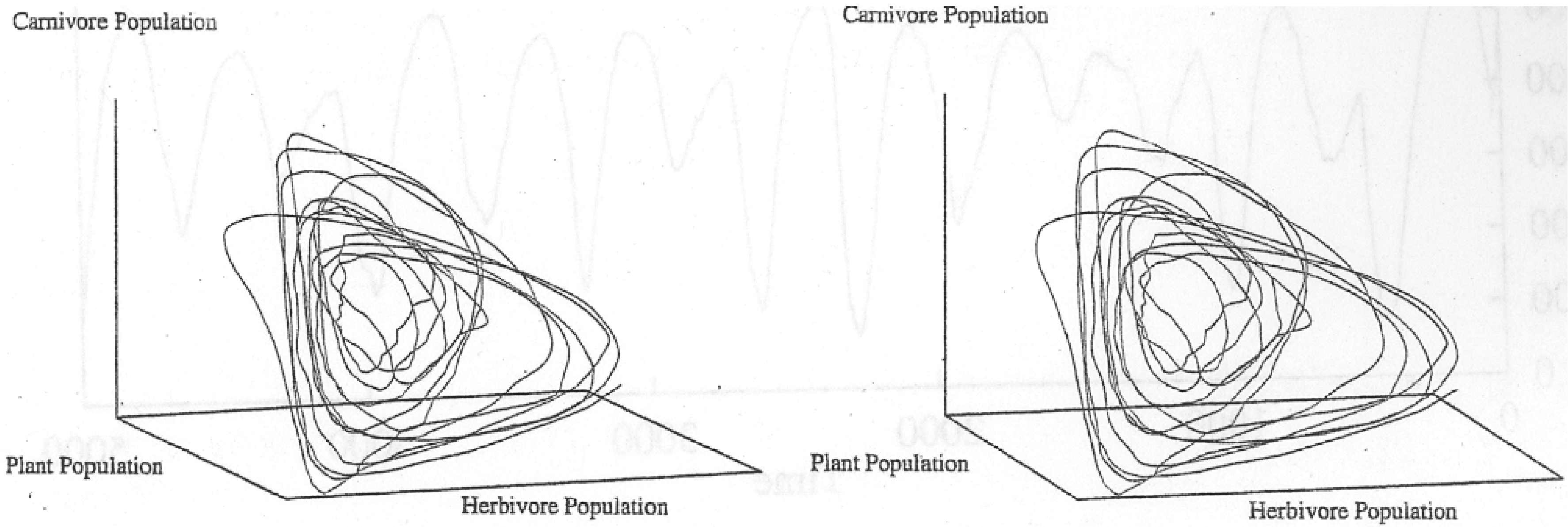


Figure 12.9 A dual-image stereogram of the attractor of the individual-based predator-prey system: To view, stare at the center of the two images and cross your eyes until the two images merge. Allow your eyes to relax so that they can refocus.

There is structure with some order mixed with disorder.

Conclusion

- Chaos is order masquerading as disorder.
- Systems tend to approach chaos from two directions:
  - The simple model produces complex behavior.
  - A complex model settles down into a behavior described by simple (three variable) model.
- We have simplicity yielding complexity and complexity yielding simplicity.
- Different phenomena can be described with similar mathematical tools because producer-consumer type interactions are common in different areas.
- Instead of microscopic or macroscopic viewpoints, the intermediate scales order and disorder balance out to produce interesting behavior.

Thank You